of inventory-queue model to evaluate the inventory cost and service level achievable for the given inventory control policy, and then found a very simple algorithm to find an optimal inventory control policy that minimizes the overall inventory holding cost and satisfies the given service level requirements.

Azad et al. (2008) presented a complex distribution network design problem in the supply chain system which included location and inventory decisions. Customers' demandswere generated randomly and each distribution center maintained a certain amount of safety stock to achieve a certain service level for the customers.Since the modelwasin a non-linear integer programming mode,the researchers proposed a hybrid heuristic tabu search with simulated annealing (SA) sharing the same tabu list developed for solving the problem.In another study, Taleizadeh et al. (2008) investigated a stochastic replenishment multi-product inventory model and proposed two models for two cases of uniform and exponential time distribution between two replenishments. They showed that the models were integer-nonlinear programming problems and developed a SA algorithm to solve them.

Alfa et al. (2008)presenteda discrete time GI[x]/G[y]/1 queuing system. To do so, some general results were obtained about the stability condition, stationary distributions of the queue lengths and waiting times. In addition, a GI/M/1 type Markov chain associated with the age process of the customers in service was also developed. Hill (2007) also investigated continuous-review lost-sales inventory models with no fixed order cost and a Poisson demand process. The objective of the study, which included a holding cost per unit per time unit and a lost sales cost per unit, was to minimize the long-run total cost and explore alternative approaches which might offer better solutions. Kiesmuller et al. (2006)studies a single node in a supply chain that faced stochastic demand. They investigated the waiting time in an (R,s,Q) inventory system under compound renewal demand. At the end, they provided an approximation for the distribution function of the customer waiting time and determined the minimal reorder level subject to the maximum average waiting time. Dong et al. (2005) developed a network of inventory-queue models for the performance modeling and analysis of an integrated logistic network.Thestudy extended the previous work done on the supply network model with base-stock control and service requirements. Instead of one-for-one base stock policy, batch-ordering policy and lot-sizing problems were considered in the study. Moreover, as in practice the assumption of incapacitated production often does notholdtrue, $GI^X/G/1$ queuing analysis was used to replace the $M^X/G/\infty$ queue based method. In addition, to include the lot-sizing issue in the analysis of stores, a fixed-batch target-level production authorization mechanism was employed to explicitly obtain performance measures of the logistic chain queuing model.

Maiti et al. (2005)proposed a deterministic inventory model of a damageable item with variable replenishment rate and unit production cost. In the study,the replenishment rate and unit production cost were dependent on demand while demand and damageability were stock- dependent; the dependency could be linear or non-linear. The optimum inventory level was evaluated by the profit maximization principle through an SA algorithm. Arslan et al. (2001) proved the optimal inventory policy structure for both continuous and discrete-time M/G/1 and G/M/1 models with an alternate source of goods and make-to-order productions. They also provided an expression from which inventory costs could be calculated for an M/M/1 model although no closed-form expression for the optimal policies was possible. Gallien et al. (2001) examined the component procurement problem in a single-item, make-to-stock assembly system. The suppliers were incapacitated and had independent but non-identically distributed stochastic delivery lead times. The assembly was instantaneous, the product demand followed a Poisson process, and the unsatisfied demand was backordered. The aim of the studywas to minimize the sum of steady state holding and backorder costs over a pre-specified class of replenishment policies. Combining the existing results ofthe queuing theory with the original results concerning distributions that are closed under maximization and translation,the researchersoffered a simple approximate solution for the problem when lead time variances were identical.

Since the proposed model is a non-linear integer mathematical programming and then is overly Np-hard, utilizing meta-heuristic algorithms to solve itis one of the best ways. In this respect, many meta-heuristic algorithms such as genetic algorithm, simulated annealing (Pasandideh et al., 2011), particle swarm optimization (Poli, 2007; Hajipour and Pasandideh, 2012), Tabu search (Zarrinpoor and Seifbarghy, 2011)are proposed. Nowadays, it is quite common to develop new meta-heuristic algorithms and apply them to various optimization problems. As an example, Taleizadeh et al. (2011) proposed a multiproduct inventory control problem in which the periods between the two replenishments of the products wereconsidered independent random variables, and increasing and decreasing functions were assumed to model the dynamic demands of each product. Furthermore, the quantities of the orders wereregardedeas integer-type, space and budget were constraints, the service-level was a chance-constraint, and the partial back-ordering policy was taken into account for the shortages. Besides, the costs of the problem were holding, purchasing, and shortage. Having considered all these conditions, the researchers presented a harmony search algorithm (introduced by Geem, 2001) to solve the model.

Recently, a new meta-heuristic algorithm named imperialist competitive algorithm (ICA) was developed by Atashpaz-Gargari and Lucas (2007). The proposers of the algorithm drew inspiration from the socio-political evolution of human. The suitability of this algorithm is demonstrated in some problems such as flow shop scheduling (Behnamian and Zandieh 2011), Game theory (Rajabioun et al., 2008), integrated product mix-outsourcing problem (Nazari-Shirkouhi et al. 2010), K-means data clustering (Niknam et

Q_{wj} The order quantity of the warehouse for product *j*
h_{rj} The holding cost rate at the retailer for product j
A_{rj} The fixed cost of ordering related to the retailer for product *j*
T_{rj} The time interval between two consecutive orders of the retailer for product *j*
φ_j The arrival rate of the customer for product *j*
μ_j The service rate of the server for product *j*
ρ_j The productivity coefficient of product *j*
$\bar{I}_j$ The average inventory level at the retailer between (0,*T*) during the lost sales period for product *j* which is equivalent to the queue length for product *j*
π_j The fixed shortage cost of product *j*
L_j The length of lead time of product *j* is assumed to be constant
F The available warehouse space for the retailer in all products
f_j The space occupied by each unit of product *j*
G The number of allowed shortage
P_j The service level for product *j*
S The expected allowable shortage cost in lost sales state
Γ_j The maximum inventory in the warehouse for product *j*
SS_j The safety stock for product *j*
Q_{rj} The stockpile amount random variable in the batch arrival queuing system for product *j* which is equivalent to the order quantity of the retailer for product *j*
m_j The coefficient of the retailer's order quantity into the warehouse
$E[Q_{rj}]$ The average stockpile amount of product *j*
y_{1j} The random demand in period *T* for product *j* which acts as Poisson distribution $y_{1j} \sim pp(\lambda_{1j})$

y_{2j} The random demand in period *L* for product *j* which acts as Poisson distribution $y_{2j} \sim pp(\lambda_{2j})$

$y_j=(y_1+y_2)_j$ The random demand in period *L+T* for product *j* which acts as Poisson distribution with parameter $\lambda_j=\lambda_{1j}+\lambda_{2j}$

R_j The maximum inventory position after order for product *j*
$P(y_j)$ The demand probability density function

$\bar{b}(R_j)$ The average shortage for product *j*
ECH_r The expected holding cost per time unit at the retailer in the steady state
ECL_r The expected shortage cost per time unit at the retailer in the steady state in lost sales state
ETC_r The expected total cost per time unit at the retailer in the steady state
ETC_w The expected total cost per time unit at the warehouse in the steady state
ETC_B The expected total system (retailer and warehouse) cost per time unit in the steady state
$k_1(R,T)$The expected total cost per time unit at the retailer in the steady state in the (*R,T*) system

It should be mentioned that the demand in the (*R,T*) system in the period *L+T* is as

$$D_{rL+T} \sim pp(\lambda_1+\lambda_2) \quad (1)$$

$$L_r+T_r \sim \text{Erlang}(2,\lambda) \quad (2)$$

In order to formulate the mathematical model, firstthe main parameters of the proposed model should be determined. The expected total cost per time unit at the retailer which includes the ordering, holding and shortage costs in the (R,T) system is as follows(Vijayan and Kumaran, 2008):

$$k_1(R,T)=\frac{1}{Tr}A + h\bar{I} + \frac{\pi}{Tr}\bar{b}(R) \quad (3)$$

Since *T* has an exponential distribution under the assessment system, we use $\frac{1}{E[Tr]}$ instead of $\frac{1}{Tr}$ to determine $\bar{b}(R_j)$ in Eq. (4).

$$\bar{b}(R_j) = \sum_{yj=Rj}^{\infty} P(yj).(yj - Rj) \quad (4)$$

Where $P(y_j)$ are as

$$P(y_j)=P(N(L+T)=n)=\lambda e^{-\lambda n}(n+\frac{1}{\lambda}) \quad (5)$$

Now, we can calculate $\bar{b}(R_j)$ as follows:

$$\bar{b}(R_j) = \sum_{yj=Rj}^{\infty} \lambda j e^{-\lambda j yj}(yj + \frac{1}{\lambda j}).(yj - Rj) \quad (6)$$

With regard to this subject and the periodic order system as the inventory order policy, the average number of customers in the system for a long time can be considered to calculate the average inventory in the periodic order inventory system. Therefore, the average number of customers in the system for a long time in the$M^{Qr}/M/1$system is (Pirayesh and Haji, 2007):

$$\bar{I}_j=\frac{\rho j}{1-\rho j}+\frac{\rho j(\frac{E[Q_{rj}^2]-1}{E[Qrj]})}{2(1-\rho j)}+\bar{b}(R_j) \quad (7)$$

where$\rho_j = \frac{\varphi j E[Qrj]}{\mu j}$.It should be pointed out that in steady state,$\varphi j E[Qrj] < \mu j$should be observed. Since the batch size is a random variable withthe Poisson distribution per *T* time,$E[Q_{rj}]$ and $E[Q_{rj}^2]$are respectively obtained from Eq. (8) and Eq. (9) as follows:

$$E[Q_{rj}]= \sum_{i=0}^{Rj} \sum_{Qrj=Rj-1}^{\infty} (Rj-1) \times \frac{e^{-\lambda 1 j} \lambda 1 j^{Qrj}}{Qrj!} \quad (8)$$

$$E[Q_{rj}^2]= \sum_{i=0}^{Rj} \sum_{Qrj=Rj-1}^{\infty} (Rj-1)^2 \times \frac{e^{-\lambda 1 j} \lambda 1 j^{Qrj}}{Qrj!} \quad (9)$$

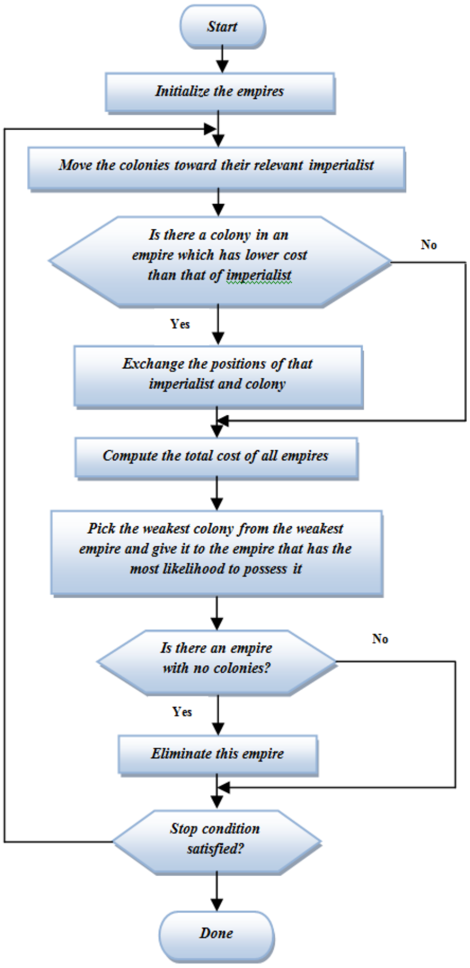

Fig. 2. Flow chart of the proposed ICA

3.1.1. Generating initial countries

In this subsection, an array of decision variable values is formed to determine the optimal values in the search area. In the ICA, the number of countries ($N_{country}$), imperialists (N_{imp}), and colonies (N_{col}) should be determined. The relationship between these algorithm parameters is $N_{country} = N_{imp} + N_{col}$.

In the ICA optimization, the aforementioned array is called '*country*'. A country is an $1 \times N$ array which is defined as:

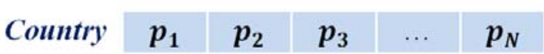

Fig. 3. A country structure

Where p_N is the normalized power of imperialist N which is defined as Eq. (19). In fact, the normalized power of an imperialist is determined by the proportion of colonies it possesses. Besides, to form initial empires, all colonies should be assigned to the imperialists based on their power. The normalized cost of an imperialist is, therefore, computed as Eq. (20).

$$p_t = \left|\frac{C_t}{\sum_{t=1}^{N_{imp}} C_t}\right|; t = 1,2,\ldots,N_{imp} \quad (19)$$

$$C_t = a_t - \max_i\{a_i\} \quad (20)$$

where a_t is the cost of the t^{th} imperialist and C_t is its normalized cost. Obviously, the colonies will be randomly chosen the size of $N_{col} \times p_t$, and then will be assigned to the imperialists. To clarify the process of initialization phase, we show it schematically in Figure4.

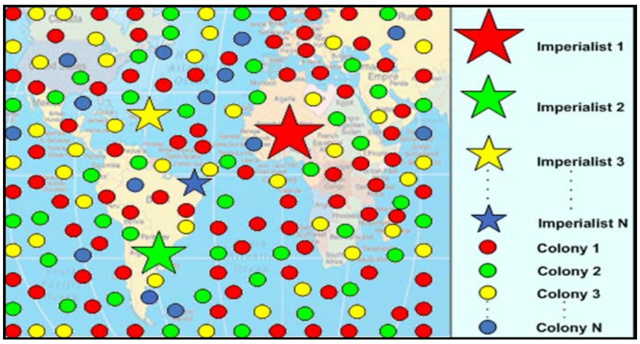

Fig. 4. A scheme of initialized empires and colonies (Atashpaz-Gargariand Lucas, 2007)

3.1.2. Movement of the colonies

After initializing the countries and selecting the empires, the imperialist countries attempt to enhance the number of their colonies by moving all the colonies toward the imperialists. This process is carried out by generating a random variable (x) which acts as the uniform distribution $X \sim Uniform(0, \beta \times d); \beta > 1$ (Atashpaz-Gargari and Lucas, 2007) where the parameter d indicates the distance between a colony and an imperialist. To explore variations around an imperialist, the concept of the deviation of a path (θ)which acts as the uniform distribution $\theta \sim U(-\gamma, \gamma)$ (Atashpaz-Gargariand Lucas, 2007) is necessary. In this concept, the parameter γ is defined as the deviation from the original direction.

It is worthy to mention that while moving toward an imperialist, a colony may reach a position with lower costs in comparison with the imperialists. In such cases, the imperialist moves to the position of that colony and vice versa. In the ICA, this process is called 'exchanging positions of the imperialist and a colony'.

3.1.3. Empires power evaluation

In this step, the total power of each empire is calculated as the sum of the imperialist cost and the average of colonies cost as follows:

$$TP_t = Cost(\text{imperialist}_t)$$
$$+[rand() \times mean\{Cost(colonies\ of\ \text{empire}_t)\}] \quad (21)$$

functions into three groups: (I) smaller-the-better type, (II) larger-the-better type, and (III) nominal-is-the-best type. As almost all the objective functions in the inventory control problem are classified as the smaller-the-better type, the corresponding S/N ratio (SNR) is as follows(Peace, 1993):

$$\text{SNR}= -10 \log(objective\ function)^2 \quad (26)$$

In order to apply the Taguchi method, firstly the levels of all the parameters should be determined (Tables 1 and 2).It should be noted that according to the sensitivity of the factor to the problem size, we determine the best value of them separately.

Table 1
Parameters levels of the proposed ICA for different problems

Factor	Symbol	Problem	Level (1)	Level (2)	Level (3)
Beta	A	All Problems	1	1.25	1.5
Sigma	B	All Problems	0.05	0.3	0.6
Lambda	C	All Problems	0.2	0.3	0.6
MaxScapeAngle	D	All Problems	8	12	16
(Npop, Nimp)	E	Problem 1	(20,2)	(30,2)	(70,3)
		Problem 2	(30,3)	(40,3)	(80,4)
		Problem 3	(40,4)	(50,4)	(90,5)
		Problem 4	(50,5)	(60,5)	(100,5)
$MaxIT_{ICA}$	F	Problem 1	20	40	60
		Problem 2	30	50	70
		Problem 3	40	60	80
		Problem 4	50	70	90

Table 2
Parameters levels of the proposed SA for different problems

Factor	Symbol	Problem No.	Level (1)	Level (2)	Level (3)	Level (4)
T_0	A	All Problems	800	1200	1600	2000
$MaxIT_{SA}$	B	Problem 1	50	90	130	170
		Problem 2	60	100	140	180
		Problem 3	70	110	150	190
		Problem 4	80	120	160	200

The Taguchi designs for the SA and ICA are $L_{16}(4^2)$ and $L_{27}(3^6)$, respectively. Tables 3 and 4 report the SNRs and the mean ratiosofthe four experimental problems based on the ICA and SA.

With regard to the outputs of the MINITAB software, the optimal values of the ICA and SA parameters can be determined according to the maximum SNR and minimum MEAN rules (Figures6 and 7). To do so, we provide all the optimal values in Table 4. It should be mentioned that here this process is reported for problem number 1.

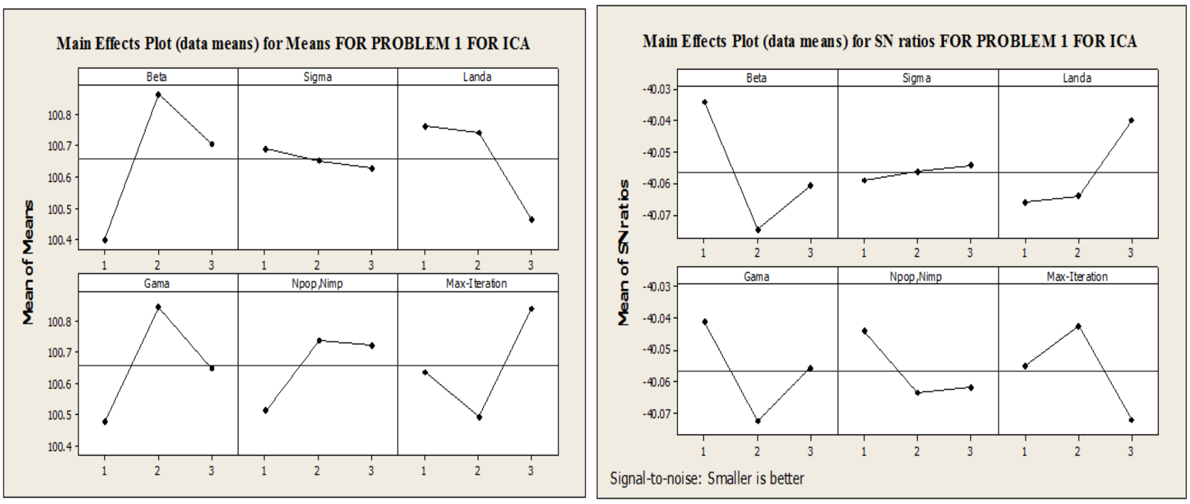

Fig. 6.The SNR and Mean ratio for the ICA in problem No. 1

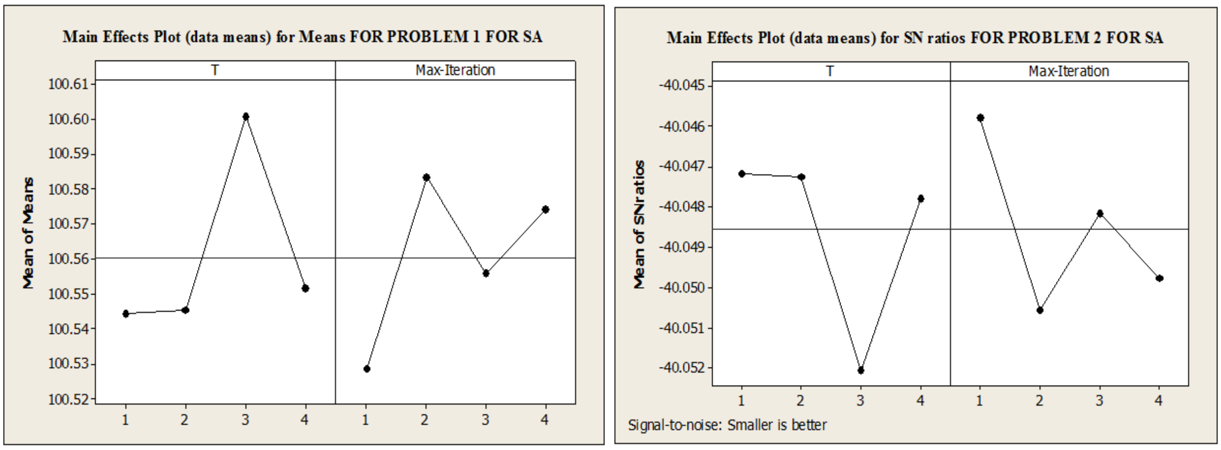

Fig. 7.The SNR and Mean ratio for the SA in problem No. 1

Table 5
Optimal values of the meta-heuristic parameters

Solving Methodologies	Factor	Optimal Value Problem 1	 Problem 2	 Problem 3	 Problem 4
ICA	Beta	1	1	1.25	1
	Sigma	0.6	0.05	0.6	0.05
	Lambda	0.6	0.3	0.6	0.3
	MaxScapeAngle	8	16	16	16
	(Npop, Nimp)	(20,2)	(30,3)	(90,5)	(60, 5)
	$MaxIT_{ICA}$	40	50	80	70
SA	T_0	800	800	800	1200
	$MaxIT_{ICA}$	50	60	150	120

5. Results and Comparisons

In this section, we first provide our four numerical examples in Table 6. Then to demonstrate the performance of the proposed algorithms, we analyze the results statistically and graphically. For each numerical example, 10 independent runs are performed by the proposed ICA and SA to decrease the uncertainty of generated runs. The reported value is based on the algorithms outputs in these 10 runs provided in Table 7. The first column of Table 7 indicates the problem number (according to the number of products) and the 2th-7th columns show the number of runs for the RPD, MIN, and TIME criteria. The experimental tests of this study were carried out on a personal computer with a Pentium processor (1.86 GHz) and one GB RAM, and the algorithms were coded by MATLAB (Version 7.10.0.499, R2010a).

The algorithms outputs are compared with each other in the following terms:

(I) Relative percentage deviation (RPD): This criterion is well-developed for measuring the efficiency of mathematical programming models. The RPD is obtained as Eq. (27):

$$RPD=[(MIN_{stage}-MIN_{total})/\ MIN_{total}]\times 100\% \quad (27)$$

where MIN_{stage} and MIN_{total} are the best cost of the algorithm in each stage and the best cost that it has had up to now, respectively. Obviously, the algorithms with the lowest RPD are the best.

(II) Best cost (MIN): The algorithms with better objective functions are the best ones.

(III) Computational time (TIME): The computational time of running the algorithm to reach the best solutions.

The outputs of all the criteria for each problem are reported in Table 8.In order to compare the algorithms, we run a T-paired statistical analysis at the 95% confidence level. Finally, to determine the best solving methodologies, based on the role of accepting H_0 hypotheses, the value of test statistic must be in the acceptance region $[-t_{\alpha,n-1},+\infty]$ or $P-value>\alpha$. The statistical analysis and comparisons are done by MINITAB and summarized in Table 9. According to the Table 19, the proposed ICA significantly works better than the SA in terms of the RPD and MIN criteria. Yet, proposed SA shows a better performance based on the Time criterion. To clarify these results, graphical comparisons are illustrated in Figs.8 and 9.

6. Conclusion and Suggestions for Future Research

In this study, a multi-product continues review inventory control problem within the batch arrival queuing approach ($M^{Qr}/M/1$) was formulated to determine the optimal quantities of maximum inventory. The objective function was to minimize the summation of ordering, holding and shortage costs under the warehouse space, service level, and the expected lost-sales shortage cost constraints from the retailer and warehouse viewpoints. Since the proposed model is Np-Hard, an efficient imperialist competitive algorithm (ICA) was proposed to solve it. To justify the proposed ICA, a simulated annealing algorithm (SA) was used to demonstrate the applicability of the proposed ICA. Moreover, a parameter tuning procedure was followed to find the best outputs of the algorithm. The results showed that the proposed ICA significantly works better than the SA in terms of the RPD and MIN criteria while the

Table 8
Computational results of the SA and ICA for all the problems

Algorithms	Criterion	Problem1	Problem2	Problem3	Problem4
SA	RPD	0.00614	0.01179	0.12920	0.262778
ICA		0.00414	0.01231	0.03033	0.015962
SA	MIN	100.571	332.97	522.618	646.72
ICA		100.371	333.14	471.841	520.32
SA	TIME	3.682	10.190	15.197	31.044
ICA		3.888	12.302	59.094	132.478

Table 9
Statistical analyses of all the criteria

Criterion	***Test Statistic***	***P-value***	***Result***
RPD	$T-\text{value}=1.49<t_{0.05,9}=1.833$	$0.884>0.05$	$D\geq 0; RPD_{SA}>RPD_{ICA}$
Min	$T-\text{value}=1.48<t_{0.05,9}=1.833$	$0.883>0.05$	$D\geq 0; MIN_{SA}>MIN_{ICA}$
TIME	$T-\text{value}=-1.55<t_{0.05,9}=1.833$	$0.109>0.05$	$D<0; TIME_{SA}<TIME_{ICA}$

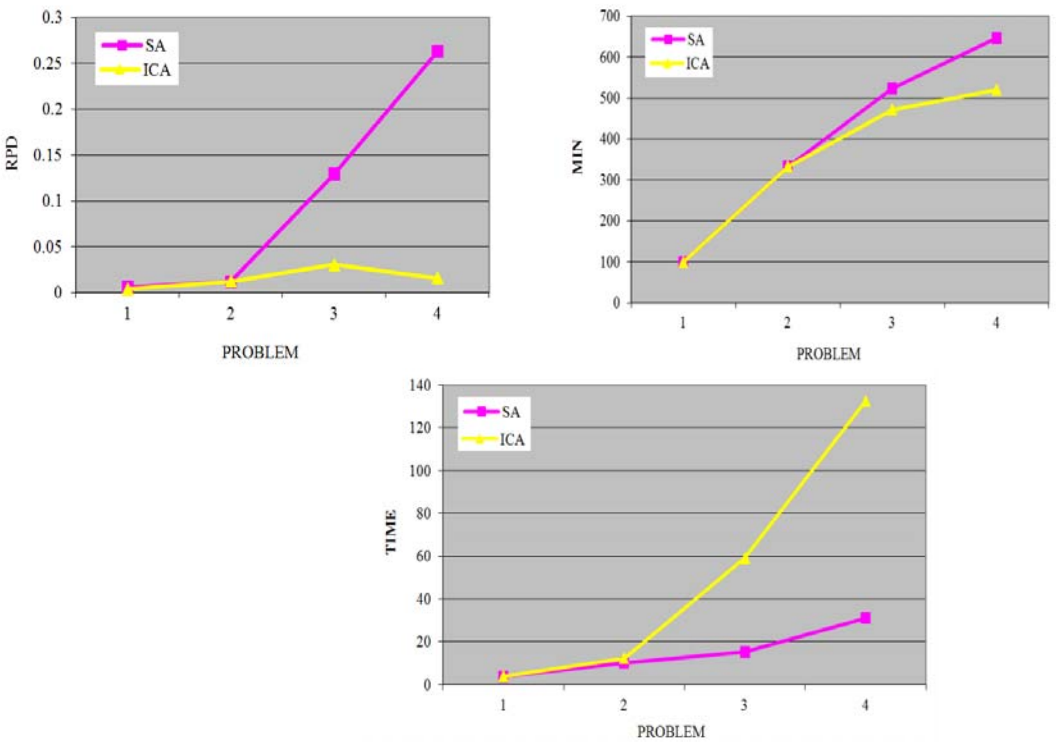

Fig. 8. Graphical comparisons of the SA and ICA based on all the criteria for all the problems

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