A Benders Decomposition Method to Solve an Integrated Logistics Network Designing Problem with Multiple Capacities

Nadieh Sodagari [a], Ahmad Sadeghi [b,*]

[a] MSc, Faculty of Industrial & Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran

[b] Assistant Professor, Faculty of Industrial & Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran

Received 11 January, 2014; Revised 15 March, 2015; Accepted 16 March, 2015

Abstract

In this paper, a new model is proposed for the integrated logistics network designing problem. In many research papers in this area, it is assumed that there is only one option for the capacity of each facility in the network. However, this is not a realistic assumption because generally there may be many possible options for the capacity of the facility that is being established. Usually the cost of establishing a facility depends on its capacity. Moreover, of the majority of the research done in the field of logistics network designing problem only a limited number of options for product recovery is addressed. Specifically, in most of the research papers only one option, i.e. remanufacturing, has been considered. Therefore, a mathematical formulation with multiple options for capacities and product recovery is addressed in this research to obviate this gap. Afterwards a benders decomposition method is developed to efficiently solve the problem. The computational results introduce several random generated problems to be solved with benders algorithm and demonstrate that this algorithm can efficiently solve the proposed model.

Keywords: Logistics network designing problem; Integrated logistics; Multiple capacities; Recycling; Benders decomposition.

1. Introduction

The traditional view of many manufacturers regarding the used products is assuming that they are valueless. They generally do not feel any obligation about what happens to the product discarded by the customer. They design their products to minimize the cost of materials, assembly and distribution but do not consider the costs of repairing, reusing or recycling (Zhou & Wang, 2008). Even though reusing the products discarded by the customers is not a new subject and it has been around in some industries for a long time (Srivastava, 2008), the level of product recovery has significantly risen throughout the last decades (Fleischmann, Beullens, Bloemhof-Ruwaard, & Wassenhove, 2001) and this fact is a reminder of the necessity of reverse logistics.

The reverse logistics can be defined as "the process of planning, implementing, and controlling the efficient and cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal" (Hawks, 2006). Such a concept has been around for a long time and many researchers have investigated it from many different viewpoints, most of them developed the traditional models considering real-case applications. For example Roghanian and Pazhuheshfar (2014) solved a stochastic reverse logistics model using genetic algorithm. Also Hatefi and Julai (2014) proposed a robust logistic model under demand uncertainly and facility disruptions. Other related works can be found in Rahmati, Ahmadi, and Karimi (2014) and Mehdizade and Fatehi Kivi (2014).

Nowadays, increasing concerns for environmental issues and passing new laws to protect the environment have highlighted the importance of reverse logistics. In many industries, such as electronic products, considering reverse logistics has become a necessity, especially with continuously decreasing product life cycle in these industries. Therefore designing an efficient reverse logistics network to reuse the products that are at the end of their life cycles is of major importance. Figure 1 illustrates the structures of forward and reverse logistics.

Establishing an efficient reverse logistics requires a well-designed network with a set of activities, such as collecting, inspecting, dismantling, recycling, remanufacturing and repairing (Kannan, Pokharel, & Sasi Kumar, 2009). In order to obtain an optimum network simultaneously considering both forward and reverse is indispensable, because the design of one network affects the optimum design for the other one, therefore optimizing these networks separately will lead to sub-optimality. However, in many research papers in the field

* Corresponding Author E-mail: a.sadeghi@qiau.ac.ir

designing a supply chain network that included finding the optimal locations of facilities and distribution warehouses so that the cost of the network is minimized. This study is the first research considering multiple options for the capacity of the facilities in the network. However, this study considers a single-commodity forward supply chain network. Ko and Evans (2007) highlighted the importance of concurrently considering the forward and reverse networks and proposed an integrated multi-commodity logistics network. They constructed a non-linear integer programming model to solve this problem. Since this problem is NP-hard, they proposed a genetic algorithm to solve this problem. However, they did not consider multiple options for the capacities. They considered only one option for product recovery, Zhou and Wang (2008) also studied designing of a generic integrated logistics network considering two product recovery options in their model. They developed a mathematical model and a branch and bound method to solve this problem. However, they didn't consider any limit for the capacity of the facilities. Pishvaee, Jolai, and Razmi (2009) proposed a stochastic optimization model for an integrated logistics network under uncertainty aiming to minimize the expected value of costs. However, their study considers only one option for product recovery and the capacity of the facilities. Mutha and Pokharel (2009) proposed a mathematical model, the reverse logistics network designing problem, with the recovery options of remanufacturing, recycling and disposal. Alumur, Nickel, Saldanha-Da-Gama, and Verter (2012) proposed a multi-period reverse logistics network designing problem. They developed an integer programming model to solve this problem and suggested that their model can be used for real-world problems. They considered a multi-commodity network and aimed to maximize the profit. They also showed the advantages of their model in comparison to static models through many different scenarios. But the network considered in their model is not integrated and only one product recovery option is considered.

Some other related works can be found in the literature, where all of their contributions are to use multiple objective functions, locating facilities, considering forward and reverse models simultaneously. However, one can hardly find any research regarding multiple capacity options for facilities. Also, very few studies considered more than one option for product recovery. Therefore, because of the existing of the discussed gap, this paper introduces an integrated logistics network designing problem with multiple options for the capacities of the facilities and several options for product recovery. Because of the complexity of the proposed problem, Benders decomposition algorithm is used to solve this model. To cognize why this model is NP-hard, one can refer to Ko and Evans (2007). Therefore for providing a perceivable description of our work, the rest of this paper is organized as follows:

In section 2, the proposed model is illustrated and a mathematical formulation is proposed to solve this problem. After developing a benders decomposition method in Section 3, the model is experimented and computational results are presented in Section 4. Finally, Section 5 is assigned to conclusion remarks and also future activities.

2. Problem Description and Mathematical Formulation

In this section the discussed problem is described and a mathematical model is proposed to solve it. This part of the article is divided into two subsections where subsection 2.1 describes the preliminaries of the problem and subsection 2.2 formulates the mathematical model.

1.1 Problem description

The problem considered in this study involves managing the reverse flow in the forms of repairing, remanufacturing, recycling and disposal. There are four types of entities in the network: customers, distribution centers, central recovery centers (CRCs) and production plants. In order to reduce the costs of network Pishvaee et al. (2009) and Lee and Dang (2008) suggested that the distribution and collection facilities use the same resources for transporting materials, production, human resources and infrastructures. In the proposed problem in this study it is assumed that the customers return the used products to the hybrid distribution-collection centers and then the returned products are sent to the CRCs. The reverse flows are managed in CRCs (Srivastava, 2008). In the CRCs the returned products are inspected and assigned to perform one of the following actions: repairing, remanufacturing, disposing, and recycling. According to Thierry et al (1995) these actions are defined as:

Repairing: the objective is to restore the products to a working condition. The quality of the repaired products is generally lower than the brand new products.

Remanufacturing: The objective is to enhance the quality of the used product to reach the standard of a brand new product. In this process the used products are completely dismantled and all of its components are inspected.

Recycling: The objective is to use the raw material of the used product. In recycling the identity of the product is lost.

In the proposed problem, the repairable products are repaired in the CRCs and sent back to the hybrid distribution-collection centers. The products that are assigned to be remanufactured, are sent to the production plants and after remanufacturing are sent back to the hybrid distribution-collection centers. The products that are assigned to be recycled are sent to the recycling centers and the disposable products are sent to the disposal centers. The logistics network proposed in this paper have 6 layers including manufacturing/remanufacturing plants, hybrid

m that is repaired at CRC k through distribution-collection center j

X^r_{mljki} Percentage of returns of customer l for product m that is remanufactured at plant i through distribution-collection center j and CRC k

X^r_{mljko} Percentage of returns of customer l for product m that is disposed at disposal center o through distribution-collection center j and CRC k

X^r_{mljkr} Percentage of returns of customer l for product m that is recycled at recycling center r through distribution-collection center j and CRC k

W_{ml} Percentage of the returns of customer l for product m that is unanswered

Y^{e}_{rh} Equals to 1 if recycling center r with capacity level h is established; Zero otherwise.

Y^{p}_{ih} Equals to 1 if manufacturing/remanufacturing plant i with capacity level h is established; Zero otherwise.

Y^{d}_{jh} Equals to 1 if hybrid distribution-collection center j with capacity level h is established; Zero otherwise.

Y^{r}_{kh} Equals to 1 if CRC k with capacity level h is established; Zero otherwise.

Y^{x}_{oh} Equals to 1 if disposal center o with capacity level h is established; Zero otherwise.

Using these notations, the problem can be formulated as follows:

$$\min \sum_{i\in I}\sum_{h\in H} f^p_{ih}y^p_{ih} + \sum_{j\in J}\sum_{h\in H} f^d_{jh}y^d_{jh} + \sum_{k\in K}\sum_{h\in H} f^r_{kh}y^r_{kh} + \sum_{o\in O}\sum_{h\in H} f^x_{oh}y^x_{oh} + \sum_{r\in R}\sum_{h\in H} f^e_{rh}y^e_{rh} + \sum_{m\in M}\sum_{i\in I}\sum_{j\in J}\sum_{l\in L} C^f_{mijl}d_{ml}X^f_{mijl}$$
$$+ \sum_{m\in M}\sum_{k\in K}\sum_{j\in J}\sum_{l\in L} C^f_{mjkl}d_{ml}X^f_{mkjl} + \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K} C^r_{mljk}r_{ml}X^r_{mljk}$$
$$+ \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{i\in I} C^r_{mljki}r_{ml}X^r_{mljki} + \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{o\in O} C^r_{mljko}r_{ml}X^r_{mljko}$$
$$+ \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{r\in R} C^r_{mljkr}r_{ml}X^r_{mljkr} + \sum_{m\in M}\sum_{l\in L} C^u_{ml}d_{ml}U_{ml} + \sum_{m\in M}\sum_{l\in L} C^w_{ml}r_{ml}W_{ml} \quad (1)$$

S.T.

$$\sum_{l\in L}\sum_{j\in J} X^r_{mljk}r_{ml} = \sum_{l\in L}\sum_{j\in J} X^f_{mkjl}d_{ml} \qquad \forall m \in M, k \in K \quad (2)$$
$$\sum_{i\in I}\sum_{j\in J} X^f_{mijl} + \sum_{k\in K}\sum_{j\in J} X^f_{mkjl} + U_{ml} = 1 \qquad \forall m \in M, l \in L \quad (3)$$
$$\sum_{j\in J}\sum_{k\in K}\left(\sum_{i\in I} X^r_{mljki} + \sum_{o\in O} X^r_{mljko} + \sum_{r\in R} X^r_{mljkr} + X^r_{mljk}\right) + W_{ml} = 1 \quad \forall m \in M, l \in L \quad (4)$$
$$\sum_{j\in J}\sum_{k\in K}\sum_{l\in L} X^r_{mljki}r_{ml} \le \sum_{j\in J}\sum_{l\in L} X^f_{mijl}d_{ml} \qquad \forall m \in M, i \in I \quad (5)$$
$$\gamma_m\left(X^r_{mljk} + \sum_{i\in I} X^r_{mljki} + \sum_{o\in O} X^r_{mljko} + \sum_{r\in R} X^r_{mljkr}\right) \le \sum_{o\in O} X^r_{mljko} \quad \forall m \in M, l \in L, j \in J, k \in K \quad (6)$$
$$\beta_m\left(X^r_{mljk} + \sum_{i\in I} X^r_{mljki} + \sum_{o\in O} X^r_{mljko} + \sum_{r\in R} X^r_{mljkr}\right) \ge X^r_{mljk} \quad \forall m \in M, l \in L, j \in J, k \in K \quad (7)$$
$$\eta_m\left(X^r_{mljk} + \sum_{i\in I} X^r_{mljki} + \sum_{o\in O} X^r_{mljko} + \sum_{r\in R} X^r_{mljkr}\right) \ge \sum_{r\in R} X^r_{mljk} \quad \forall m \in M, l \in L, j \in J, k \in K \quad (8)$$
$$\sum_{j\in J} X^f_{mijl}d_{ml} \le \sum_{h\in H} cap^p_{ih}Y^p_{ih} \qquad \forall m \in M, l \in L, i \in I, l \in L \quad (9)$$
$$\sum_{j\in J} X^f_{mijl}d_{ml} + \sum_{k\in k} X^f_{mkjl}d_{ml} \le \sum_{h\in H} cap^d_{jh}Y^d_{jh} \qquad \forall m \in M, l \in L, j \in J \quad (10)$$
$$\sum_{j\in J} X^r_{mljk}r_{ml} + \sum_{j\in J}\sum_{i\in I} X^r_{mljki}r_{ml} + \sum_{j\in J}\sum_{o\in O} X^r_{mljko}r_{ml} + \sum_{j\in J}\sum_{r\in R} X^r_{mljkr}r_{ml} \le \sum_{h\in H} cap^r_{kh}Y^r_{kh}$$
$$\forall m \in M, k \in K, l \in L \quad (11)$$
$$\sum_{k\in K} X^r_{mljk}r_{ml} + \sum_{k\in K}\sum_{i\in I} X^r_{mljki}r_{ml} + \sum_{k\in K}\sum_{o\in O} X^r_{mljko}r_{ml} + \sum_{k\in K}\sum_{r\in R} X^r_{mljkr}r_{ml} \le cap^d_{rj}Y^d_k$$
$$\forall m \in M, j \in J, l \in L \quad (12)$$
$$\sum_{j\in J}\sum_{k\in K} X^r_{mljki}r_{ml} \le \sum_{h\in H} cap^p_{rih}Y^p_{rih} \qquad \forall m \in M, i \in I, l \in L \quad (13)$$
$$\sum_{j\in J}\sum_{k\in K} X^r_{mljko}r_{ml} \le \sum_{h\in H} cap^x_{roh}Y^x_{oh} \qquad \forall m \in M, i \in I, l \in L \quad (14)$$
$$\sum_{j\in J}\sum_{k\in K} X^r_{mljkr}r_{ml} \le \sum_{h\in H} cap^e_{rh}Y^e_{rh} \qquad \forall m \in M, i \in I, l \in L \quad (15)$$
$$0 \le X^f_{mijl}, X^f_{mkjl}, X^r_{mljki}, X^r_{mljk}, X^r_{mljko}, U_{ml}, X_{ml} \le 1 \quad \forall m \in M, i \in I, j \in J, k \in K, r \in R, l \in L \quad (16)$$
$$Y^p_i, Y^d_j, Y^r_k, Y^x_o, Y^e_r \in \{0,1\} \quad \forall m \in M, i \in I, j \in J, k \in K, r \in R, l \in L \quad (17)$$

The introduced model is a modified one which is based on the past researches, except that all constraints related to the capacity options shape the contribution of our work. On the other hand, traditional models are improved to contain the contributions.

In this model constraint (1) shows the objective function which is minimizing the total cost of the logistics network. Constraint (2) ensures that all of the repaired products are used to meet the demands of the customers.

$X^r_{mljko}, X^r_{mljkr}, U_{ml}, W_{ml}$) for the fixed values of $(\hat{y}^p_{ih}, \hat{y}^d_{jh}, \hat{y}^r_{kh}, \hat{y}^x_{oh}, \hat{y}^e_{rh})$. This problem can be developed as follows:

$$\min \sum_{m\in M}\sum_{i\in I}\sum_{j\in J}\sum_{l\in L} C^f_{mijl} d_{ml} X^f_{mijl} + \sum_{m\in M}\sum_{k\in K}\sum_{j\in J}\sum_{l\in L} C^f_{mkjl} d_{ml} X^f_{mkjl} + \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K} C^r_{mljk} r_{ml} X^r_{mljk} + \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{i\in I} C^r_{mljki} r_{ml} X^r_{mljki} + \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{o\in O} C^r_{mljko} r_{ml} X^r_{mljko} + \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{r\in R} C^r_{mljkr} r_{ml} X^r_{mljkr} + \sum_{m\in M}\sum_{l\in L} C^u_{ml} d_{ml} U_{ml} + \sum_{m\in M}\sum_{l\in L} C^w_{ml} r_{ml} W_{ml} \tag{18}$$

$$\sum_{l\in L}\sum_{j\in J} X^r_{mljk} r_{ml} \le \sum_{l\in L}\sum_{j\in J} X^f_{mkjl} d_{ml} \qquad \forall m \in M, k \in K \tag{19}$$

$$\sum_{l\in L}\sum_{j\in J} X^r_{mljk} r_{ml} \ge \sum_{l\in L}\sum_{j\in J} X^f_{mkjl} d_{ml} \qquad \forall m \in M, k \in K \tag{20}$$

$$\sum_{i\in I}\sum_{j\in J} X^f_{mijl} + \sum_{k\in K}\sum_{j\in J} X^f_{mkjl} + U_{ml} \le 1 \qquad \forall m \in M, l \in L \tag{21}$$

$$\sum_{i\in I}\sum_{j\in J} X^f_{mijl} + \sum_{k\in K}\sum_{j\in J} X^f_{mkjl} + U_{ml} \ge 1 \qquad \forall m \in M, l \in L \tag{22}$$

$$\sum_{j\in J}\sum_{k\in K}\Big(\sum_{i\in I} X^r_{mljki} + \sum_{o\in O} X^r_{mljko} + \sum_{r\in R} X^r_{mljkr} + X^r_{mljk}\Big) + W_{ml} \le 1 \quad \forall m \in M, l \in L \tag{23}$$

$$\sum_{j\in J}\sum_{k\in K}\Big(\sum_{i\in I} X^r_{mljki} + \sum_{o\in O} X^r_{mljko} + \sum_{r\in R} X^r_{mljkr} + X^r_{mljk}\Big) + W_{ml} \ge 1 \quad \forall m \in M, l \in L \tag{24}$$

$$\sum_{j\in J}\sum_{k\in K}\sum_{l\in L} X^r_{mljki} r_{ml} \le \sum_{j\in J}\sum_{l\in L} X^f_{mijl} d_{ml} \qquad \forall m \in M, i \in I \tag{25}$$

$$\gamma_m\left(X^r_{mljk} + \sum_{i\in I} X^r_{mljki} + \sum_{o\in O} X^r_{mljko} + \sum_{r\in R} X^r_{mljkr}\right) \le \sum_{o\in O} X^r_{mljko} \quad \forall m \in M, l \in L, j \in J, k \in K \tag{26}$$

$$\beta_m\left(X^r_{mljk} + \sum_{i\in I} X^r_{mljki} + \sum_{o\in O} X^r_{mljko} + \sum_{r\in R} X^r_{mljkr}\right) \ge X^r_{mljk} \quad \forall m \in M, l \in L, j \in J, k \in K \tag{27}$$

$$\eta_m\left(X^r_{mljk} + \sum_{i\in I} X^r_{mljki} + \sum_{o\in O} X^r_{mljko} + \sum_{r\in R} X^r_{mljkr}\right) \ge \sum_{r\in R} X^r_{mljkr} \quad \forall m \in M, l \in L, j \in J, k \in K \tag{28}$$

$$\sum_{j\in J} X^f_{mijl} d_{ml} \le \sum_{h\in H} cap^p_{ih} \hat{Y}^p_{ih} \qquad \forall m \in M, l \in L, i \in I, \tag{29}$$

$$\sum_{i\in I} X^f_{mijl} d_{ml} + \sum_{k\in k} X^f_{mkjl} d_{ml} \le \sum_{h\in H} cap^d_{jh} \hat{Y}^d_{jh} \qquad \forall m \in M, l \in L, j \in J \tag{30}$$

$$\sum_{j\in J} X^r_{mljk} r_{ml} + \sum_{j\in J}\sum_{i\in I} X^r_{mljki} r_{ml} + \sum_{j\in J}\sum_{o\in O} X^r_{mljko} r_{ml} + \sum_{j\in J}\sum_{r\in R} X^r_{mljkr} r_{ml} \le \sum_{h\in H} cap^r_{kh} \hat{Y}^r_{kh} \qquad \forall m \in M, k \in K, l \in L \tag{31}$$

$$\sum_{k\in K} X^r_{mljk} r_{ml} + \sum_{k\in K}\sum_{i\in I} X^r_{mljki} r_{ml} + \sum_{k\in K}\sum_{o\in O} X^r_{mljko} r_{ml} + \sum_{k\in K}\sum_{r\in R} X^r_{mljkr} r_{ml} \le \sum_{h\in H} cap^d_{rjh} \hat{Y}^d_{jh} \qquad \forall m \in M, j \in J, l \in L \tag{32}$$

$$\sum_{j\in J}\sum_{k\in K} X^r_{mljki} r_{ml} \le \sum_{h\in H} cap^p_{rih} \hat{Y}^p_{rih} \qquad \forall m \in M, i \in I, l \in L \tag{33}$$

$$\sum_{j\in J}\sum_{k\in K} X^r_{mljko} r_{ml} \le \sum_{h\in H} cap^x_{oh} \hat{Y}^x_{oh} \qquad \forall m \in M, o \in O, l \in L \tag{34}$$

$$\sum_{j\in J}\sum_{k\in K} X^r_{mljkr} r_{ml} \le \sum_{h\in H} cap^e_{rh} \hat{Y}^e_{rh} \qquad \forall m \in M, r \in R, l \in L \tag{35}$$

$$0 \le X^f_{mijl}, X^f_{mkjl}, X^r_{mljki}, X^r_{mljk}, X^r_{mljko}, U_{ml}, X_{ml} \le 1 \quad \forall m \in M, i \in I, j \in J, k \in K, r \in R, l \in L \tag{36}$$

$$-\sum_{m\in M}\sum_{l\in L}\hat{\pi}^{3l'}_{ml}+\sum_{m\in M}\sum_{l\in L}\hat{\pi}^{4l'}_{ml}-\sum_{m\in M}\sum_{l\in L}\hat{\pi}^{5l'}_{ml}+\sum_{m\in M}\sum_{l\in L}\hat{\pi}^{6l'}_{ml}$$
$$-\sum_{m\in M}\sum_{l\in L}\sum_{i\in I}\sum_{h\in H}cap^{p}_{ih}\hat{Y}^{p}_{ih}\hat{\pi}^{11l'}_{mli}-\sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{h\in H}cap^{d}_{jh}\hat{Y}^{d}_{jh}\hat{\pi}^{12l'}_{mlj}$$
$$-\sum_{m\in M}\sum_{l\in L}\sum_{k\in K}\sum_{h\in H}cap^{r}_{kh}\hat{Y}^{r}_{kh}\hat{\pi}^{13l'}_{mkl}-\sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{h\in H}cap^{d}_{rjh}\hat{Y}^{d}_{jh}\hat{\pi}^{14l'}_{mjl}$$
$$-\sum_{m\in M}\sum_{l\in L}\sum_{i\in I}\sum_{h\in H}cap^{p}_{rih}\hat{Y}^{p}_{ih}\hat{\pi}^{15l'}_{mil}-\sum_{m\in M}\sum_{l\in L}\sum_{o\in O}\sum_{h\in H}cap^{x}_{oh}\hat{Y}^{x}_{oh}\hat{\pi}^{16l'}_{mol}$$
$$-\sum_{m\in M}\sum_{l\in L}\sum_{r\in R}\sum_{h\in H}cap^{e}_{rh}\hat{Y}^{e}_{rh}\hat{\pi}^{17l'}_{mrl}\le 0\quad \forall l'=1,\dots,\acute{L} \tag{49}$$

$$\sum_{h}y^{p}_{ih}\le 1\ \forall i \tag{50}$$

$$\sum_{h}y^{d}_{jh}\le 1\ \forall j \tag{51}$$

$$\sum_{h}y^{r}_{kh}\le 1\ \forall k \tag{52}$$

$$\sum_{h}y^{x}_{oh}\le 1\ \forall o \tag{53}$$

$$\sum_{h}y^{e}_{rh}\le 1\ \forall r \tag{54}$$

In this model constraint (46) is the objective function of benders master problem. Constraint (47) is the optimality cut which is introduced to the master problem if the sub-problem is solved to optimality. Parameters $\hat{\pi}^{1k'}_{mk}$ ‘$\hat{\pi}^{2k'}_{mk}$ ‘ $\hat{\pi}^{3k'}_{ml}$ ‘$\hat{\pi}^{4k'}_{ml}$ ‘$\hat{\pi}^{5k'}_{ml}$ ‘$\hat{\pi}^{6k'}_{ml}$ ‘$\hat{\pi}^{7k'}_{mi}$ ‘$\hat{\pi}^{8k'}_{mljk}$ ‘$\hat{\pi}^{9k'}_{mljk}$ ‘$\hat{\pi}^{10k'}_{mljk}$ ‘$\hat{\pi}^{11k'}_{mli}$ ‘ $\hat{\pi}^{12k'}_{mlj}$ ‘$\hat{\pi}^{13k'}_{mkl}$ ‘$\hat{\pi}^{14k'}_{mjl}$ ‘$\hat{\pi}^{15k'}_{mil}$ ‘$\hat{\pi}^{16k'}_{mol}$ ‘$\hat{\pi}^{17k'}_{mrl}$ are the values of dual variables obtained by solving benders dual sub-problem. Constraint (48) is the feasibility cut which is added to the master problem, that is the sub-problem, which is infeasible.

3.1.2 Overall procedure of benders decomposition method

The pseudo code for the overall procedure of Benders decomposition is presented in Fig. 2.

As it is shown in this figure, the procedure starts with an initial feasible solution for the master problem. This can be done by solving the problem without any additional cuts. Then the obtained solution for the master problem is given to the sub-problem, if the sub-problem is infeasible, i.e. the dual sub-problem is unbounded, an unbounded ray is used to generate an infeasibility cut to add to the master problem for the next iteration. If the sub-problem is feasible and solved to optimality, using the optimal solution obtained, an optimality cut is generated and added to the master problem for the next iteration. If the obtained solution provides a better upper bound, the upper bound is updated. Then the master problem is solved again. This process is repeated until the gap between the lower bound and upper bound is lower than a specified value.

This algorithm is developed in GAMS 23.1 and used to solve a numerical instance of the problem and the result is compared to the mathematical model, which is also solved using GAMS 23.1. The results are presented in Table 1. According to this table solving the mathematical model directly require 83.04 seconds to obtain the optimal solution, but using the benders decomposition method, this time can be reduced to 17.428 seconds. Considering these times, one can conclude that the differences between them are not admissible. However, increasing the size of the problem will lead to a greater gap for times and in such situation, the proposed benders will outperform the traditional solver.

Fig. 3 shows the upper bound and lower bounds obtained by Benders decomposition method in different iterations. As you can see in this figure, the algorithm reaches the optimal solution after 11 iterations.

4. Computational Results

In this section several instances with different number of products and hybrid distribution-collection centers are solved using the benders decomposition method. Then managerial insights are discussed. The considered instance is limited to 6 candidate locations for manufacturing/remanufacturing plants, 10 candidate locations for hybrid distribution-collection centers, 7 candidate locations for CRCs, 2 candidate locations for disposal centers and 3 candidate locations for recycling centers, 20 customer points and two products.

4.1 Single commodity network

In this section an instance of the problem with a single product is considered. The data for this instance is generated randomly and the problem is solved using the proposed benders decomposition method. After 28

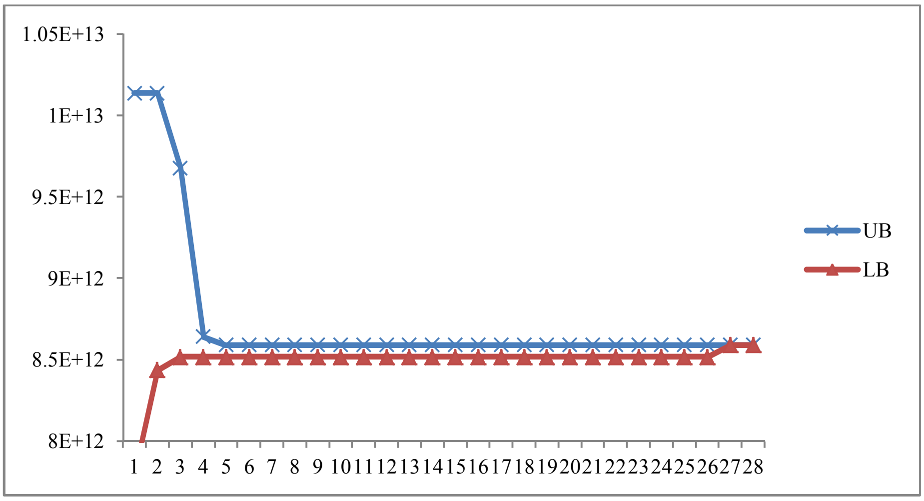

Fig. 4. Convergence of the algorithm for the single commodity network

Table 2
Results for the single commodity network

Solution method	Running time (s)	Objective function
Mathematical model	51.141	8589342671353
Benders Decomposition	33.075	8589342671353

The percentage of demands of the customers that are satisfied by different manufacturing plants and CRCs is presented in Fig. 5. In this figure the horizontal axis shows the customers while the vertical axis shows the percentage of demand of each customer that is satisfied by a manufacturer or CRC. The percentage of returns retrieved by different manufacturing plants, disposal centers, recycling centers and CRCs is presented in Fig. 6.

4.2 Multi-commodity network

In this section an instance of the problem with multiple products is considered. The data for this instance is generated randomly and the problem is solved using the proposed benders decomposition method. After 17 iterations of the algorithms, the optimal cost of the network is obtained as 17178626108393 monetary units. The convergence of the algorithm for this instance is presented in Fig. 7. Moreover, in order to compare the results of the proposed method and mathematical model, their results for this instance is presented in Table 3.

In order to further investigation, the distribution of the satisfied demand and returns, the percentages of the demands and returns for different products are presented in Figures 8 through 11.

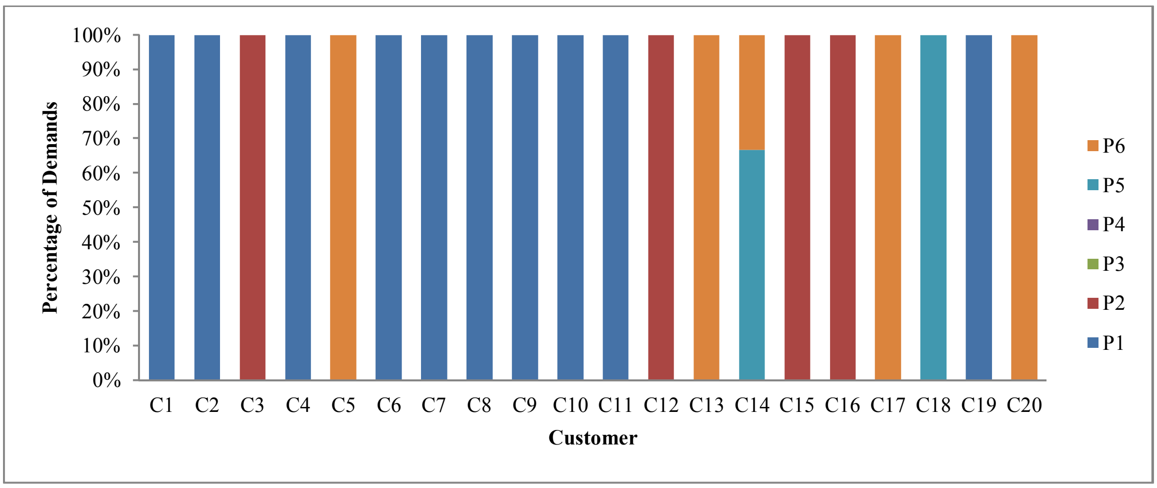

Fig. 5. Percentage of demands satisfied by manufacturing plant and CRCs

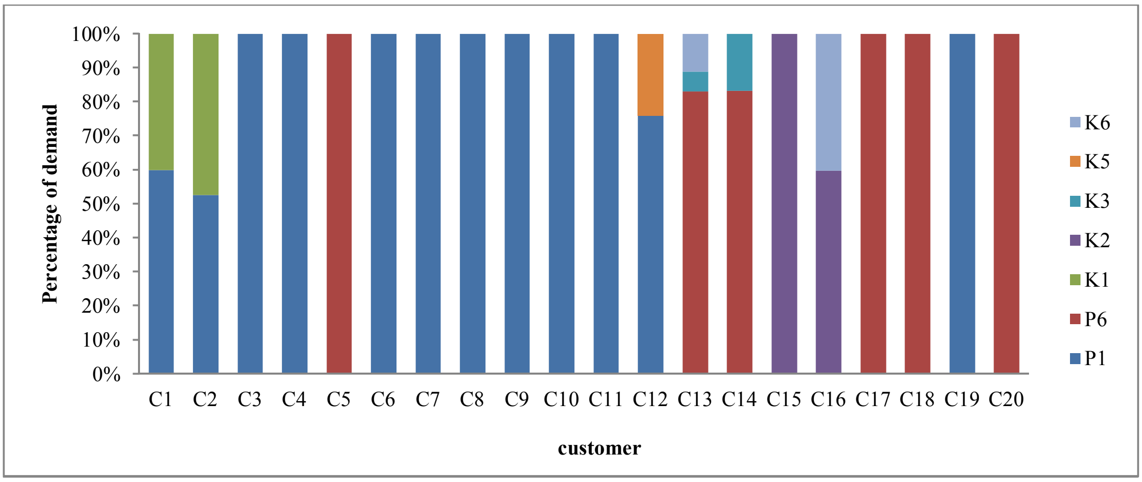

Fig. 8. Percentage of demands satisfied by manufacturing plant and CRCs for product 1

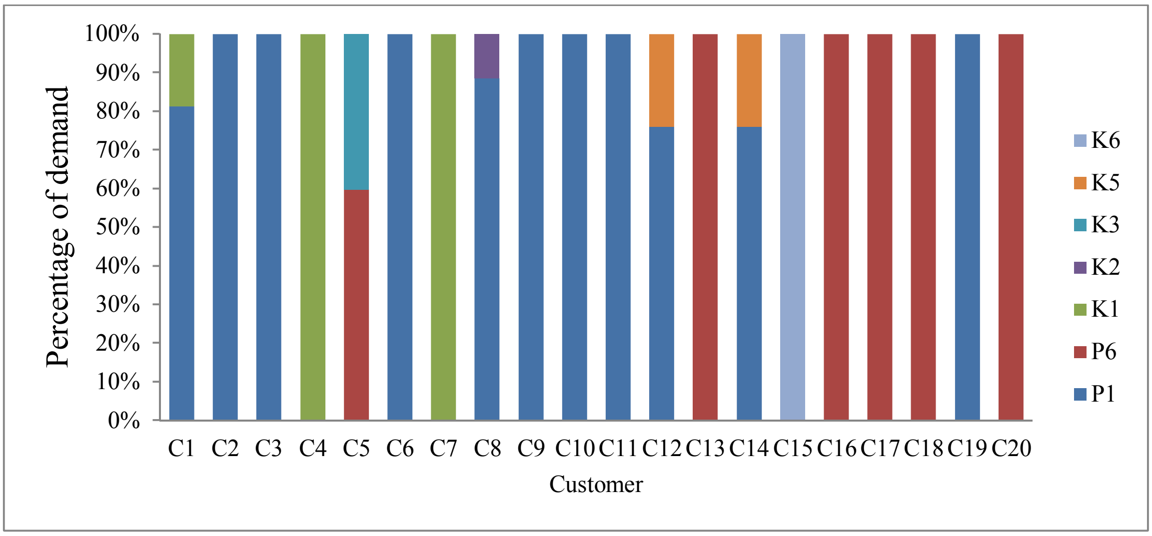

Fig. 9. Percentage of demands satisfied by manufacturing plant and CRCs for product 2

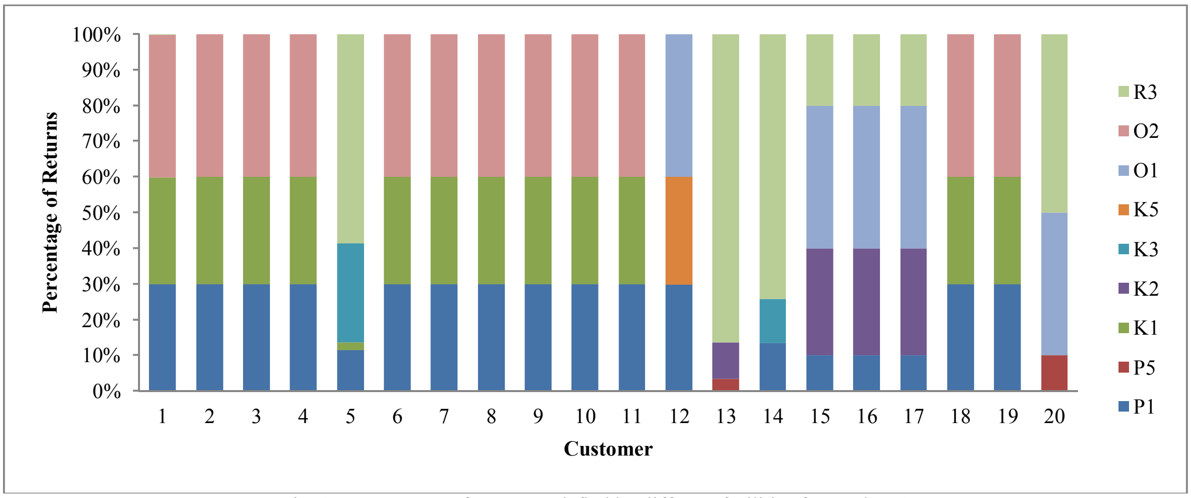

Fig. 10. Percentage of returns satisfied by different facilities for product 1

selection of reverse logistics provider' Resources, Conservation and Recycling, Vol.54, pp. 28-36.
[12] Lee, D.-H. & Dong, M. (2008) 'A heuristic approach to logistics network design for end-of-lease computer products recovery' Transportation Research Part E: Logistics and Transportation Review, Vol.44, pp. 455-474.
[13] Meade, L. & Sarkis, J. (2002) 'A conceptual model for selecting and evaluating third-party reverse logistics providers' Supply Chain Management: An International Journal, Vol.7, pp. 283-295.
[14] Mehdizadeh, E. & Fatehi Kivi, A. (2014) 'Three Metaheuristic Algorithms for Solving the Multi-item Capacitated Lot-sizing Problem with Product Returns and Remanufacturing' Journal of Optimization in Industrial Engineering, Vol.7, pp. 41-53.
[15] Melo, M. T., Nickel, S. & Saldanha-Da-Gama, F. (2009) 'Facility location and supply chain management – A review' European Journal of Operational Research, Vol.196, pp. 401-412.
[16] Pishvaee, M. S., Jolai, F. & Razmi, J. (2009) 'A stochastic optimization model for integrated forward/reverse logistics network design' Journal of Manufacturing Systems, Vol.28, pp. 107-114.
[17] Pokharel, S. & Mutha, A. (2009) 'Perspectives in reverse logistics: A review' Resources, Conservation and Recycling, Vol.53, pp. 175-182.
[18] Rahmati, H. A., Ahmadi, A. & Karimi, B. (2014) 'Facility Location and Inventory Balancing in a Multi-period Multi-echelon Multi-objective Supply Chain: An MOEA Approach' Journal of Optimization in Industrial Engineering, Vol.7, pp. 83-99.
[19] Realff, M. J., Ammons, J. C. & Newton, D. J. (2004) 'Robust reverse production system design for carpet recycling' IIE Transactions, Vol.36, pp. 767-776.
[20] Roghanian, E. & Pazhuheshfar, P. (2014) 'An optimization model for reverse logistics network under stochastic environment by using genetic algorithm' Journal of Manufacturing Systems, Vol.33, pp. 348-356.
[21] Srivastava, S. K. (2008) 'Network design for reverse logistics' Omega, Vol.36, pp. 535-548.
[22] Thierry, M. C., Salomon, M., Nunen, J. V. & Wassenhove, L. V. (1995) 'Strategic issues in product recovery management' California management review, Vol.37, pp. 114-135.
[23] Wang, S. J., Hung, H. M. & O`Neill, R. (2011) 'Adaptive design clinical trials and trial logistics models in CNS drug development' European Neuropsychopharmacology, Vol.21, pp.159-166.
[24] Yeh, W.-C. (2005) 'A hybrid heuristic algorithm for the multistage supply chain network problem' The International Journal of Advanced Manufacturing Technology, Vol.26, pp. 675-685.
[25] Zhou, Y. & Wang, S. (2008) 'Generic model of reverse logistics network design' Journal of Transportation Systems Engineering and Information Technology, Vol.8, pp. 71-78.