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A New Dynamic Random Fuzzy DEA Model to Predict Performance of Decision Making Units

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Abstract

Data envelopment analysis (DEA) is a methodology for measuring the relative efficiency of decision making units (DMUs) which consume the same types of inputs and producing the same types of outputs. Assuming that future planning and predicting the efficiency are very important for DMUs, this paper first presents a new dynamic random fuzzy DEA model (DRF-DEA) with common weights (using multi objective DEA approach) to predict the efficiency of DMUs under mean chance constraints and expected values of the objective functions. In the initial proposed DRF-DEA model, the inputs and outputs are assumed to be characterized by random triangular fuzzy variables with normal distribution, in which data are changing sequentially. Under this assumption, the solution process is very complex. So we then convert the initial proposed DRF-DEA model to its equivalent multi-objective stochastic programming, in which the constraints contain the standard normal distribution functions, and the objective functions are the expected values of functions of normal random variables. In order to improve in computational time, we then convert the equivalent multi-objective stochastic model to one objective stochastic model with using fuzzy multiple objectives programming approach. To solve it, we design a new hybrid algorithm by integrating Monte Carlo (MC) simulation and Genetic Algorithm (GA). Since no benchmark is available in the literature, one practical example will be presented. The computational results show that our hybrid algorithm outperforms the hybrid GA algorithm which was proposed by Qin and Liu (2010) in terms of runtime and solution quality.

Keywords: Stochastic Data envelopment analysis; Dynamic programming; random fuzzy variable; Monte Carlo simulation; Genetic algorithm.

1. Introduction

Data envelopment analysis is an important managerial tool for evaluating and improving the performance of decision making units in systems. Data envelopment analysis (DEA) which was initially proposed by Charnes, Cooper and Rhodes (1978) has been widely applied to evaluate the relative efficiency of a set of DMUs based on multiple criteria. Since the first DEA model (CCR), it has been surveyed by researchers very quickly to various areas. The advantage of this technique is that it does not require the explicit specification of functional relations between the multiple inputs and outputs or either a priori weights. However, when we measure the efficiency of DMUs, the data in traditional DEA models are often limit to crisp data and the efficiency scores of DMUs are very sensitive to data variations and don't allow the stochastic variations in the data, such as data entry errors and measurement errors. With considering stochastic variations in outputs and inputs, a DMU which is efficient relative to other DMUs may convert to be inefficient. Cooper, Huang and Li (1996) is the first one who developed a stochastic DEA model with chance constrained programming reflecting theories of behavior in social psychology. Usually we obtain fuzzy data from DMUs, because various experts may have various ideas, especially one expert may have different ideas during different times. Sengupta (1992) is the first one who considered fuzziness both in objective and constraints and analyzed the fuzzy DEA model. Entani, Maeda and Tanaka (2002) changed fuzzy input and output data in to intervals by using α-level sets, and suggested two different interval DEA models. Since in real world problems, decision makers may encounter a hybrid uncertain environment where randomness and fuzziness coexist in a decision system, they represent the inputs and outputs in these systems by random fuzzy variables to characterize the hybrid uncertainty. Recently, the random fuzzy variable (Kwakernaak, 1978), possibility theory (Z.Q. Liu and Y.K. Liu, 2010), credibility theory and

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(2008) presented an equivalence model between MOLP and dynamic DEA models and present how a dynamic DEA problem can be solved interactively without any prior judgments by transforming it into an MOLP formulation. Fukuyama (2012) presented a dynamic network DEA model with common weights to measure productivity change for 269 Japanese Shinkin banks during 2002 to 2009. Omrani (2013) introduced a robust optimization approach to find common weights in DEA with uncertainty in data. Ramezani and Khodabakhshi (2013) proposed model to ranking DMUs with using common weights set in DEA in dynamic environment. The aim of this study is to show the criteria used by Wong. Wang, Lu and Liu (2014) proposed a new multi-objective two stage fuzzy DEA model in dynamic environment for evaluating the performance of US bank holding companies. This model provides a set of common weights for comparing performance and increases the discriminating power. Kawaguchi, Tone and Tsutsui (2014) estimated the efficiency of Japanese hospitals using a dynamic network DEA model with common weights. Tone and Tsutsui (2014) proposed a new slacks-based measure approach in dynamic DEA with network structure and common weights set. It can use for input oriented model and analysis efficiency variations in network. Tavana et al. (2015) presented a common set of weights (CSW) model for ranking the DMUs with the stochastic data and the ideal point concept. The proposed method minimizes the distance between the evaluated DMUs and the ideal DMU. Hatami-Marbini et al. (2015) proposed a DEA model for centrally imposed resource or output reduction. They used a common set of weights method for controlling the weight flexibility and reducing the computational complexities.

The crisp outputs and inputs in traditional DEA models become random fuzzy variables in fuzzy stochastic environment, and modeling with such data is meaningless directly because the meanings of the constraints and the objective are not clear at all. In fact, we have faced such situation in fuzzy and stochastic environment, in which we deal with the fuzzy data and random data with credibility and probability, respectively, to obtain a meaningful model. In fuzzy stochastic programming, the mean chance plays the same role as credibility in fuzzy environments and probability role in stochastic environments (Y. Liu and B. Liu, 2005). So, in order to obtain a meaningful model in fuzzy stochastic environments and predict the efficiencies of DMUs, we employ the expected value to objective functions and the mean chance to constraints with given confidence levels to propose a new random fuzzy DEA model with common weights in dynamic environment. In general, the mean chance functions in the constraints are difficult to compute, so we can convert the mean chance constraints to their equivalent stochastic representations according to the formula for the mean chance function (in section 3). To compute the objective functions, under the assumption that the inputs and outputs are random triangular fuzzy vectors, we present the equivalent stochastic representation of the objective functions. As a consequence, the initial proposed model can be converted to its equivalent stochastic programming one. Since the objective functions are the mathematical expectation of functions of the normal random variables, we cannot solve it via the conventional optimization algorithm. To overcome the difficulty, we combine MC simulation technique and genetic algorithm to solve it.

3. Preliminaries

3.1. Credibility approach

Let ξ be a fuzzy variable with a possibility distribution function μ. The credibility of a fuzzy event $\{\xi \geq r\}$ for $r \in R$ is defined as (Qin and Liu, 2010):

$$Cr\{\xi \geq r\} = \frac{1}{2}(1 + \sup_{t \geq r} \mu(t) - \sup_{t < r} \mu(t)) \tag{1}$$

Also "Cr" has the following property:

$$Cr\{\xi \geq r\} + Cr\{\xi < r\} = 1 \tag{2}$$

And the expected value of random fuzzy variable (ξ) is defined as:

$$E[\xi] = \int_0^{\infty} Cr\{\xi \geq r\}dr - \int_{-\infty}^{0} Cr\{\xi \leq r\}dr \tag{3}$$

Let ξ be an n-dimensional random fuzzy vector, and B a Borel subset of R. The mean chance of a random fuzzy event $\{\xi \in B\}$ is defined as:

$$Ch\{\xi \in \beta\} = \int Cr\{\xi \in \beta\} P\{\xi \in \beta\} \tag{4}$$

And the expected value of random fuzzy variable (ξ) is defined as (Qin and Liu, 2010):

$$E[\xi] = \int_0^{\infty} Ch\{\xi \geq r\}dr - \int_{-\infty}^{0} Ch\{\xi \leq r\}dr \tag{5}$$

3.2. Mean chance distributions for random triangular fuzzy variables

This section establishes some useful formulas for the mean chance functions of random triangular fuzzy variables, which will be used in the next section.

Theorem 3.2.1 Let $\xi = (X - a, X, X + b)$ be a continuous random triangular fuzzy variable, in which X is a random variable, and a,b being positive numbers. If $X \sim N(\mu, \sigma^2)$ then we have:

$$Max: \quad Z_0 = \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}}$$

$$st: \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \qquad j = 1,2,\dots,n$$

$$u_r, v_i \ge \varepsilon \qquad i = 1,\dots,m \quad r = 1,\dots,s \tag{10}$$

where x_{ij} and y_{rj} represent the *i*th input and *r*th output of DMU_j, respectively. u_r and v_i are the weights of the rth output and ith input, respectively. Finding a multipliers (u_r,v_i) for i=1,..,m and r=1,..s is the basic opinion of the efficiency measurement of the CCR model, so that the efficiency ratio in objective function can be maximized for DMU0. Generally, CCR (10) has been applied in static environment in which data supposed to be fixed during evaluation period. But in dynamic environment, it is assumed that there are "n" DMUs and their activities are examined in T periods (t =1,2,…, T) in which data are changing sequentially. In the *t*th period, each DMUj (j=1,..,n) uses two different groups of inputs: K^{t-1} (l dimensional vector of quasi-fixed inputs) and X^t(m dimensional vector of input variables) to produce two different groups of outputs: Y^t (r dimensional vector of output variables) and K^t (l dimensional vector of quasi-fixed outputs) (Nemoto and Goto, 2003). As shown in Fig. 1, the horizontal axis denotes the order of periods and the vertical axis indicates the order of DMUs. In this figure, the quasi-fixed outputs vector (K^t) in the *t*th period is used as the quasi-fixed or feedback inputs vector (link data) at the next (t + 1) period. As for variables mentioned above, the dynamic CCR model is built as following:

$$Max: \quad Z_0^t = \frac{\sum_{r=1}^{s} u_r^t y_{ro}^t + \sum_{l=1}^{L} \rho_l^t k_{l0}^t}{\sum_{i=1}^{m} v_i^t x_{i0}^t + \sum_{l=1}^{L} \beta_l^{t-1} k_{l0}^{t-1}}$$

$$st: \tag{11}$$

$$\frac{\sum_{r=1}^{s} u_r^t y_{rj}^t + \sum_{l=1}^{L} \rho_l^t k_{lj}^t}{\sum_{i=1}^{m} v_i^t x_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1} k_{lj}^{t-1}} \le 1 \qquad j = 1,2,\dots,n$$

$$u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1} \ge \varepsilon$$

$$i = 1,..,m \quad r = 1,..,s \quad l = 1,..,L \quad t = 1,..,T$$

Where ρ_l^t and β_l^{t-1} are the weights of *l*th quasi–fix outputs and quasi–fix inputs at period t (t=1,..,T), respectively. The dynamic CCR model (11) is usually applied to evaluate the relative efficiency of DMUs with crisp outputs and inputs. However, in real world problems, the data are often derived by statistic or given by experts according to their experience, so randomness and fuzziness may exist simultaneously in these data. In many cases, we can only obtain the possibility distributions of the inputs and outputs. Thus in this paper, we assume that the inputs and outputs are random triangular fuzzy variables with normal distributions, following as:

$$X_j^t = \begin{pmatrix} (x_{1j}^t - a_{1j}^t, x_{1j}^t, x_{1j}^t + b_{1j}^t) \\ (x_{2j}^t - a_{2j}^t, x_{2j}^t, x_{2j}^t + b_{2j}^t) \\ \vdots \\ (x_{mj}^t - a_{mj}^t, x_{mj}^t, x_{mj}^t + b_{mj}^t) \end{pmatrix} \qquad Y_j^t = \begin{pmatrix} (y_{1j}^t - c_{1j}^t, y_{1j}^t, y_{1j}^t + d_{1j}^t) \\ (y_{2j}^t - c_{2j}^t, y_{2j}^t, y_{2j}^t + d_{2j}^t) \\ \vdots \\ (y_{sj}^t - c_{sj}^t, y_{sj}^t, y_{sj}^t + d_{sj}^t) \end{pmatrix}$$

$$K_j^t = \begin{pmatrix} (k_{1j}^t - e_{1j}^t, k_{1j}^t, k_{1j}^t + f_{1j}^t) \\ (k_{2j}^t - e_{2j}^t, k_{2j}^t, k_{2j}^t + f_{2j}^t) \\ \vdots \\ (k_{lj}^t - e_{lj}^t, k_{lj}^t, k_{lj}^t + f_{lj}^t) \end{pmatrix} \qquad K_j^{t-1} = \begin{pmatrix} (k_{1j}^{t-1} - e_{1j}^{t-1}, k_{1j}^{t-1}, k_{1j}^{t-1} + f_{1j}^{t-1}) \\ (k_{2j}^{t-1} - e_{2j}^{t-1}, k_{2j}^{t-1}, k_{2j}^{t-1} + f_{2j}^{t-1}) \\ \vdots \\ (k_{lj}^{t-1} - e_{lj}^{t-1}, k_{lj}^{t-1}, k_{lj}^{t-1} + f_{lj}^{t-1}) \end{pmatrix} \tag{12}$$

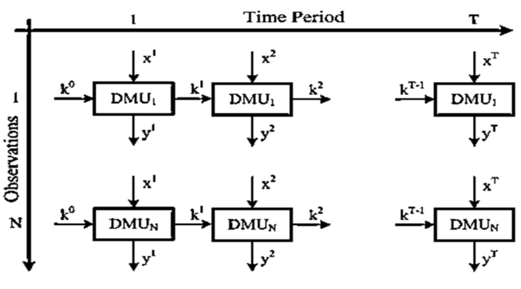

Fig. 1. Execution of the DEA model in dynamic framework

$$g_j^t(u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1}) = Ch\left\{ (\sum_{i=1}^{m} v_i^t x_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1} k_{lj}^{t-1}) - (\sum_{r=1}^{s} u_r^t y_{rj}^t + \sum_{l=1}^{L} \rho_l^t k_{lj}^t) \geq 0 \right\}$$

$$= \frac{\sqrt{\sum_{i=1}^{m} v_i^{2^t} \sigma_{ij}^{2^t} + \sum_{l=1}^{L} \beta_l^{2^{t-1}} \overline{\overline{\sigma}}_{lj}^{2^{t-1}} + \sum_{r=1}^{s} u_r^{2^t} \overline{\sigma}_{rj}^{2^t} + \sum_{l=1}^{L} \rho_l^{2^t} \overline{\overline{\sigma}}_{lj}^{2^t}}\ (\sum_{i=1}^{m} v_i^t\ (b_{ij}^t - a_{ij}^t)^2 + \sum_{l=1}^{L} \beta_l^{t-1}\ (f_{lj}^{t-1} - e_{lj}^{t-1})}{2\sqrt{2\pi}(\sum_{i=1}^{m} v_i^t\ a_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1}\ e_{lj}^{t-1} + \sum_{r=1}^{s} u_r^t\ d_{rj}^t + \sum_{l=1}^{L} \rho_l^t\ f_{lj}^t)(\sum_{i=1}^{m} v_i^t\ b_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1}\ f_{lj}^{t-1}} \cdots$$

$$\cdots \frac{-\sum_{r=1}^{s} u_r^t\ (d_{rj}^t - c_{rj}^t) - \sum_{l=1}^{L} \rho_l^t\ (f_{lj}^t - e_{lj}^t))}{+\sum_{r=1}^{s} u_r^t\ c_{rj}^t + \sum_{l=1}^{L} \rho_l^t\ e_{lj}^t)} * \exp(-\frac{(\sum_{i=1}^{m} v_i^t\ \mu_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1}\ \overline{\overline{\mu}}_{lj}^{t-1} - \sum_{r=1}^{s} u_r^t\ \overline{\mu}_{rj}^t - \sum_{l=1}^{L} \rho_l^t\ \overline{\overline{\mu}}_{lj}^t)^2}{2(\sum_{i=1}^{m} v_i^{2^t} \sigma_{ij}^{2^t} + \sum_{l=1}^{L} \beta_l^{2^{t-1}} \overline{\overline{\sigma}}_{lj}^{2^{t-1}} + \sum_{r=1}^{s} u_r^{2^t} \overline{\sigma}_{rj}^{2^t} + \sum_{l=1}^{L} \rho_l^{2^t} \overline{\overline{\sigma}}_{lj}^{2^t})})$$

$$+ \frac{\sqrt{\sum_{i=1}^{m} v_i^{2^t} \sigma_{ij}^{2^t} + \sum_{l=1}^{L} \beta_l^{2^{t-1}} \overline{\overline{\sigma}}_{lj}^{2^{t-1}} + \sum_{r=1}^{s} u_r^{2^t} \overline{\sigma}_{rj}^{2^t} + \sum_{l=1}^{L} \rho_l^{2^t} \overline{\overline{\sigma}}_{lj}^{2^t}}}{2\sqrt{2\pi}(\sum_{i=1}^{m} v_i^t\ b_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1}\ f_{lj}^{t-1} + \sum_{r=1}^{s} u_r^t\ c_{rj}^t + \sum_{l=1}^{L} \rho_l^t\ e_{lj}^t)}$$

$$* \exp(-\frac{(\sum_{i=1}^{m} v_i^t\ (b_{ij}^t + \mu_{ij}^t) + \sum_{l=1}^{L} \beta_l^{t-1}\ (f_{lj}^{t-1} + \overline{\overline{\mu}}_{lj}^{t-1}) + \sum_{r=1}^{s} u_r^t\ (c_{rj}^t - \overline{\mu}_{rj}^t) + \sum_{l=1}^{L} \rho_l^t\ (e_{lj}^t - \overline{\overline{\mu}}_{lj}^t))^2}{2(\sum_{i=1}^{m} v_i^{2^t} \sigma_{ij}^{2^t} + \sum_{l=1}^{L} \beta_l^{2^{t-1}} \overline{\overline{\sigma}}_{lj}^{2^{t-1}} + \sum_{r=1}^{s} u_r^{2^t} \overline{\sigma}_{rj}^{2^t} + \sum_{l=1}^{L} \rho_l^{2^t} \overline{\overline{\sigma}}_{lj}^{2^t})}) -$$

$$\frac{\sqrt{\sum_{i=1}^{m} v_i^{2^t} \sigma_{ij}^{2^t} + \sum_{l=1}^{L} \beta_l^{2^{t-1}} \overline{\overline{\sigma}}_{lj}^{2^{t-1}} + \sum_{r=1}^{s} u_r^{2^t} \overline{\sigma}_{rj}^{2^t} + \sum_{l=1}^{L} \rho_l^{2^t} \overline{\overline{\sigma}}_{lj}^{2^t}}}{2\sqrt{2\pi}(\sum_{i=1}^{m} v_i^t\ a_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1}\ e_{lj}^{t-1} + \sum_{r=1}^{s} u_r^t\ d_{rj}^t + \sum_{l=1}^{L} \rho_l^t\ f_{lj}^t)}$$

$$* \exp(-\frac{(\sum_{i=1}^{m} v_i^t\ (a_{ij}^t - \mu_{ij}^t) + \sum_{l=1}^{L} \beta_l^{t-1}\ (e_{lj}^{t-1} - \overline{\overline{\mu}}_{lj}^{t-1}) + \sum_{r=1}^{s} u_r^t\ (d_{rj}^t + \overline{\mu}_{rj}^t) + \sum_{l=1}^{L} \rho_l^t\ (f_{lj}^t + \overline{\overline{\mu}}_{lj}^t))^2 + \sum_{l=1}^{L} \rho_l^t\ (f_{lj}^t + \overline{\overline{\mu}}_{lj}^t))^2}{2(\sum_{i=1}^{m} v_i^{2^t} \sigma_{ij}^{2^t} + \sum_{l=1}^{L} \beta_l^{2^{t-1}} \overline{\overline{\sigma}}_{lj}^{2^{t-1}} + \sum_{r=1}^{s} u_r^{2^t} \overline{\sigma}_{rj}^{2^t} + \sum_{l=1}^{L} \rho_l^{2^t} \overline{\overline{\sigma}}_{lj}^{2^t})})$$

$$+ \frac{\sum_{i=1}^{m} v_i^t\ (\mu_{ij}^t - a_{ij}^t) + \sum_{l=1}^{L} \beta_l^{t-1}\ (\overline{\overline{\mu}}_{lj}^{t-1} - e_{lj}^{t-1}) - \sum_{r=1}^{s} u_r^t\ (\overline{\mu}_{rj}^t + d_{rj}^t) - \sum_{l=1}^{L} \rho_l^t\ (\overline{\overline{\mu}}_{lj}^t + f_{lj}^t)}{2(\sum_{i=1}^{m} v_i^t\ a_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1}\ e_{lj}^{t-1} + \sum_{r=1}^{s} u_r^t\ d_{rj}^t + \sum_{l=1}^{L} \rho_l^t\ f_{lj}^t)}$$

$$* \phi(\frac{\sum_{i=1}^{m} v_i^t\ (a_{ij}^t - \mu_{ij}^t) + \sum_{l=1}^{L} \beta_l^{t-1}\ (e_{lj}^{t-1} - \overline{\overline{\mu}}_{lj}^{t-1}) + \sum_{r=1}^{s} u_r^t\ (d_{rj}^t + \overline{\mu}_{rj}^t) + \sum_{l=1}^{L} \rho_l^t\ (f_{lj}^t + \overline{\overline{\mu}}_{lj}^t)}{\sqrt{\sum_{i=1}^{m} v_i^{2^t} \sigma_{ij}^{2^t} + \sum_{l=1}^{L} \beta_l^{2^{t-1}} \overline{\overline{\sigma}}_{lj}^{2^{t-1}} + \sum_{r=1}^{s} u_r^{2^t} \overline{\sigma}_{rj}^{2^t} + \sum_{l=1}^{L} \rho_l^{2^t} \overline{\overline{\sigma}}_{lj}^{2^t}}}$$

Where $\left(\frac{Y^t+K^t}{X^t+K^{t-1}}\right)_\alpha$ is the α-cut of the fuzzy variable $\left(\frac{Y^t+K^t}{X^t+K^{t-1}}\right)$ at period t. So we have:

$$E\left[\frac{Y^t+K^t}{X^t+K^{t-1}}\right]=\frac{1}{2}\int_0^1[\frac{y^t-(1-\alpha)c^t+k^t-(1-\alpha)e^t}{x^t+(1-\alpha)b^t+k^{t-1}+(1-\alpha)f^{t-1}}+\frac{y^t+(1-\alpha)d^t+k^t+(1-\alpha)f^t}{x^t-(1-\alpha)a^t+k^{t-1}-(1-\alpha)e^{t-1}}]\quad d\alpha \tag{18}$$

$$=-\frac{1}{2}(\frac{c^t+e^t}{b^t+f^{t-1}})-\frac{1}{2(b^t+f^{t-1})}[y^t+k^t-c^t-e^t+x^t+k^{t-1}+b^t+f^{t-1}(\frac{c^t+e^t}{b^t+f^{t-1}})]*Ln(\frac{x^t+k^{t-1}}{x^t+k^{t-1}+b^t+f^{t-1}})$$
$$-\frac{1}{2}(\frac{d^t+f^t}{a^t+e^{t-1}})+\frac{1}{2(a^t+e^{t-1})}[y^t+k^t+d^t+f^t+x^t+k^{t-1}-a^t-e^{t-1}(\frac{d^t+f^t}{a^t+e^{t-1}})]*Ln(\frac{x^t+k^{t-1}}{x^t+k^{t-1}-a^t-e^{t-1}})$$

The proof of the theorem is complete.

Suppose X_j^t , Y_j^t, K_j^t and K_j^{t-1} are random triangular fuzzy vectors of DMUj in period t as (12), then we have:

$$\mu_j^t=\sum_{i=1}^m v_i^t\mu_{ij}^t \qquad \overline{\mu}_j^t=\sum_{r=1}^s u_r^t\overline{\mu}_{rj}^t \qquad \overline{\overline{\mu}}_j^t=\sum_{l=1}^L \rho_l^t\overline{\overline{\mu}}_{lj}^t \qquad \overline{\overline{\mu}}_j^{t-1}=\sum_{l=1}^L \beta_l^{t-1}\overline{\overline{\mu}}_{lj}^{t-1}$$

$$\sigma_j^{2^t}=\sum_{i=1}^m v_i^t\delta_{ij}^{2^t} \qquad \overline{\sigma}_j^{2^t}=\sum_{r=1}^s u_r^t\overline{\delta}_{rj}^{2^t} \qquad \overline{\overline{\sigma}}_j^{2^t}=\sum_{l=1}^L \rho_l^t\overline{\overline{\delta}}_{lj}^{2^t} \qquad \overline{\overline{\sigma}}_j^{2^{t-1}}=\sum_{l=1}^L \beta_l^{t-1}\overline{\overline{\delta}}_{lj}^{2^{t-1}}$$

$$a_j^t=\sum_{i=1}^m v_i^t a_{ij}^t \qquad b_j^t=\sum_{i=1}^m v_i^t b_{ij}^t \qquad c_j^t=\sum_{r=1}^s u_r^t c_{rj}^t \qquad d_j^t=\sum_{r=1}^s u_r^t d_{rj}^t$$

$$e_j^t=\sum_{l=1}^L \rho_l^t e_{lj}^t \qquad f_j^t=\sum_{l=1}^L \rho_l^t f_{lj}^t \qquad e_j^{t-1}=\sum_{l=1}^L B_l^{t-1}e_{lj}^{t-1} \qquad f_j^{t-1}=\sum_{l=1}^L \beta_l^{t-1}f_{lj}^{t-1}$$

By Theorem (5.2.1), the *j*th objective function of initial DRF-DEA model (13) has the following equivalent stochastic representation:

$$Z_j^t=E(\frac{\sum_{r=1}^s u_r^t y_{rj}^t+\sum_{l=1}^L \rho_l^t k_{lj}^t}{\sum_{i=1}^m v_i^t x_{ij}^t+\sum_{l=1}^L \beta_l^{t-1}k_{lj}^{t-1}})=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\{-\frac{1}{2}(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})-\frac{1}{2}(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})+\frac{1}{2(b_j^t+f_j^{t-1})}[y_j^t+k_j^t+c_j^t+e_j^t+x_j^t(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})$$
$$+k_j^{t-1}(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})+b_j^t(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})+f_j^{t-1}(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})]*Ln(\frac{x_j^t+k_j^{t-1}}{x_j^t+k_j^{t-1}+b_j^t+f_j^{t-1}})+\frac{1}{2(a_j^t+e_j^{t-1})}[y_j^t+d_j^t+k_j^t+f_j^t+x_j^t(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})$$
$$+k_j^{t-1}(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})-a_j^t(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})-e_j^{t-1}(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})]*Ln(\frac{x_j^t+k_j^{t-1}}{x_j^t+k_j^{t-1}-a_j^t-e_j^{t-1}})\}*\frac{1}{4\pi^2\sigma_j^t\overline{\sigma}_j^t\overline{\overline{\sigma}}_j^t\overline{\overline{\sigma}}_j^{t-1}}\exp(-\frac{(x_j^t-\mu_j^t)^2}{2\sigma_j^{2^t}}-\frac{(y_j^t-\overline{\mu}_j^t)^2}{2\overline{\sigma}_j^{2^t}}$$
$$-\frac{(k_j^t-\overline{\overline{\mu}}_j^t)^2}{2\overline{\overline{\sigma}}_j^{2^t}}-\frac{(k_j^{t-1}-\overline{\overline{\mu}}_j^{t-1})^2}{2\overline{\overline{\sigma}}_j^{2^{t-1}}})\,dk_j^{t-1}dk_j^t dy_j^t dx_j^t \quad =\frac{1}{\sigma_j^t\sqrt{2\pi}}\int_{-\infty}^{+\infty}f(x_j^t)\quad dX_j^t-\frac{1}{2}(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})-\frac{1}{2}(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}}) \tag{19}$$

where:

$$f(x_j^t)=\{\frac{1}{2(b_j^t+f_j^{t-1})}[\overline{\mu}_j^t+\overline{\overline{\mu}}_j^t+c_j^t+e_j^t+x_j^t(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})+\overline{\overline{\mu}}_j^{t-1}(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})+b_j^t(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})+f_j^{t-1}(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})]$$
$$*Ln(\frac{x_j^t+\overline{\overline{\mu}}_j^{t-1}}{x_j^t+\overline{\overline{\mu}}_j^{t-1}+b_j^t+f_j^{t-1}})+\frac{1}{2(a_j^t+e_j^{t-1})}[\overline{\mu}_j^t+d_j^t+\overline{\overline{\mu}}_j^t+f_j^t+x_j^t(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})+\overline{\overline{\mu}}_j^{t-1}(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})$$
$$-a_j^t(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})-e_j^{t-1}(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})]*Ln(\frac{x_j^t+\overline{\overline{\mu}}_j^{t-1}}{x_j^t+\overline{\overline{\mu}}_j^{t-1}-a_j^t-e_j^{t-1}})\}*\exp(-\frac{(x_j^t-\mu_j^t)^2}{2\sigma_j^{2^t}})$$

in Z_j^t are approximated by MC simulation which will discribe in section 6.5. If R^t satisfies the constraints of model (22), then it is feasible and take it as an initial chromosome. Repeat this process until pop-size initial feasible chromosomes $R_1^t, R_2^t, ...R_{pop-size}^t$ are produced.

6. 3. Recombination process

Firstly by crossover operator, renew the chromosomes $R_p^t, p = 1,2,...,pop-size$. For crossover operation, repeat the following process to determine the parents for choromosomes from p = 1 to pop-size: generate a random real number r from the unit interval [0, 1]. If $r \prec P_c$, the chromosome R_p^t will be selected as a parent choromosome, where the parameter P_c is the probability of crossover. Then group the selected parent choromosomes $R_1^{t,1}, R_2^{t,1}, R_3^{t,1},...$ to the pairs $(R_1^{t,1}, R_2^{t,1}), (R_3^{t,1}, R_4^{t,1}),...$.The crossover process on each pair $(R_p^{t,1}, R_{p+1}^{t,1})$ is showed as follows. Generate a random number λ from the interval (0,1), then the crossover operator on $R_p^{t,1}$ and $R_{p+1}^{t,1}$ will generate two offspring $R_p^{t,2}$ and $R_{p+1}^{t,2}$ as following:

$$\begin{aligned} R_p^{t,2} &= \lambda R_p^{t,1} + (1-\lambda) R_{p+1}^{t,1} \\ R_{p+1}^{t,2} &= (1-\lambda) R_p^{t,1} + \lambda R_{p+1}^{t,1} \end{aligned} \tag{23}$$

If both offsprings satisfy the constraints of model (22), then we replace the parents with them. Otherwise, we keep the feasible one if it exists, and repeat the crossover by reproduce another real number from interval (0,1) until two feasible offsprings are obtained. Finally, we get pop-size chromosomes, including the new generated chromosomes. Secondly, update the chromosomes R_p^t by mutation operator. Repeat the following steps from p = 1 to pop-size: produce a random real number r from the [0,1], if $r \prec P_m$ the chromosome R_p^t will be selected as a parent choromosome, where the parameter P_m is the probability of mutation. For each selected parent $R_p^{t,1}$, the mutation operation is as the following:

$$R_p^{t,2} = R_p^{t,1} + r.M \tag{24}$$

where M is an appropriate large positive number. If $R_p^{t,2}$ is infeasible, then we set M as a random number between 0 and M until it is feasible. We set M = 0, if the above process cannot find a feasible solution in a predetermined number of iterations. Anyway, we replace the parent $R_p^{t,1}$ with its offspring $R_p^{t,2}$. Finally, we get pop-size chromosomes, including the new generated chromosomes.

6. 4. Evolution process

We compute the fitness of each choromosome in each period via formula (19). The chromosomes $R_1^t, R_2^t, ...R_{pop-size}^t$ are assumed to have been rearranged from good to bad according to their ftnesses. Select the chromosomes for a new population, in which the chromosome with higher fitness will have a big chance for selection. After this process pop-size times, we obtain pop-size of chromosomes, denoted also by R_p^t.

6. 5. Monte Carelo (MC) simulation

MC simulation is a method to deal with the stochastic behavior in complex systems (Chuen ,Kuan and Wai, 2012). In order to solve the final proposed DRF-DEA model (22), for any given solution $(u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1})$, we need to check its feasibility. Since the some constraints (19) include integrals $(\int_{-\infty}^{+\infty} f_j(x_j^t) \ dx_j^t)$ which cannot solve via the conventional optimization algorithm, so we should approximate their values. MC simulation method, firstly changes variable in the function $f_j(x_j^t)$ to convert infinite interval to finite interval as following:

$$\int_{-\infty}^{+\infty} f(x)dx = \int_{-1}^{+1} f\left(\frac{h}{1-h^2}\right)\frac{1+h^2}{(1-h^2)^2}dh \tag{25}$$

Also, the process of MC simulation to approximate integral (25) is described as follows:

Procedure: MC simulation for approximating (25)

begin
n←number-simulation
for i =1 to n do

hi←Generate a uniform distributed random point in the interval [a,b]=[-1,1]
Determine the average value of the function:

$$\hat{f} = \frac{1}{n} * \sum_{i=1}^{n} f\left(\frac{h_i}{1-h_i^2}\right)\left(\frac{1+h_i^2}{(1-h_i^2)^2}\right)$$

Compute the approximation to the integral:

$$\int_{-\infty}^{+\infty} f(x)dx \approx (b-a)\hat{f}$$

end

By integrating GA and MC simulation, we design a new hybrid algorithm (MC-GA) for solving the equivalent one objective stochastic model (22) which summarized as follows:

- Initialize pop_size chromosomes whose feasibility must be checked by constraints of model (22) and MC simulation.

Table 2
The predicted efficiency scores of DMUs for both periods with $\alpha = 0.5$

period	DMU	Optimal solution $(u_1^t, v_1^t, v_2^t, \rho_1^t, \beta_1^{t-1})$	α-expected efficient value
t=1	1	(0.2851 , 0.0194 , 0.9929 , 0.2521 , 0.0302)	0.927
	2	(0.3839 , 0.1708 , 0.6286 , 0.9724 , 0.0853)	0.975
	3	(0.2312 , 0.7125 , 0.7281 , 0.4173 , 0.5125)	0.884
	4	(0.5554 , 0.3531 , 0.9679 , 0.2232 , 0.8834)	0.954
	5	(0.4138 , 0.0187 , 0.8113 , 0.8266 , 0.0670)	0.837
t=2	1	(0.3151 , 0.0134 , 0.9529 , 0.1521 , 0.1302)	0.915
	2	(0.4209 , 0.1608 , 0.6086 , 0.9724 , 0.1153)	0.945
	3	(0.1981 , 0.8126 , 0.9201 , 0.4173 , 0.4824)	0.846
	4	(0.3994 , 0.4561 , 0.9709 , 0.2232 , 0.6855)	0.885
	5	(0.4935 , 0.0123 , 0.6119 , 0.8266 , 0.2685)	0.934

Table 3
The predicted efficiency scores of DMUs under various risk levels for the next two financial periods

periods	DMU	$\alpha = 0.05$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$
t=1	1	0.823	0.841	0.868	0.927	0.962
	2	0.891	0.921	0.932	0.975	0.994
	3	0.812	0.834	0.854	0.884	0.946
	4	0.827	0.849	0.871	0.954	0.973
	5	0.801	0.825	0.844	0.837	0.900
t=2	1	0.813	0.831	0.858	0.915	0.967
	2	0.888	0.918	0.927	0.945	0.995
	3	0.780	0.812	0.818	0.846	0.911
	4	0.831	0.852	0.874	0.885	0.926
	5	0.885	0.899	0.929	0.934	0.973

Table 4 documents the total amount of actual inputs and outputs for the Autumn and Winter of 2013. This study sampled these results in January 2014 in order to examine whether the predicted efficiency scores are different from actual efficiency scores. So, the results of actual efficiency scores with using conventional DEA (dynamic CCR model (11)) and actual data which are shown in Table 4 for DMUs. Generally there are three types of classification: (a) $\alpha \prec 0.5$ is conservative, (b) $\alpha = 0.5$ is risk-natural and (c) $\alpha \succ 0.5$ is risk-taking in DEA. It is easily thought that the conventional use of DEA belongs to the risk-natural (Nemoto and Goto, 2003). This finding can be easily confirmed by comparing the actual efficiency results with the predicted efficiency results under $\alpha = 0.5$. The two DEA approaches exhibit very similar results on efficiency and ranks scores. For example in second period from table 4, three DMUs (the 1th, 2th, 5th gas stations) are efficient based on actual efficiency scores and have been in the first place, while they are in the first place to third and separated based on the predicted efficiencies. Table 4 indicates that the high Pearson correlation rates have obtained (0.732 and 0.957) for both periods between predicted and actual efficiency scores with $\alpha = 0.5$.

In order to further illustrate the validation of the obtained results, we compare our results against the results of the similar hybrid GA algorithm which was proposed by Qin and Liu (2010). The computational results of the predicted efficiency scores for DMU1 are reported in Table 5, in which parameter “CPU(s)” is the computational time consumed by the two hybrid algorithms to get (near) optimal predicted efficiency score (Z_1^*). It can be from Table 5 that the proposed hybrid GA algorithm solves all instances optimally in an average of less than 61s of CPU time requirement for both periods. As a result, we conclude that our hybrid GA algorithm outperforms the Qin’s hybrid GA algorithm in terms of runtime. In addition, the predicted efficiency scores of our hybrid algorithm are closer to the actual efficiencies of computational results of the Qin’s hybrid algorithm.

$$\frac{\text{optimal } \alpha\text{-expexted efficient value - actual } \alpha\text{-expexted efficient value}}{\text{optimal } \alpha\text{-expexted efficient value}} \times 100\%$$

Table 6
Comparison solutions for DMU1 under different GA's parameters in first period under $\alpha = 0.5$

P_C	P_m	Gen	α-expected efficient value	Realative error (%)
0.1	0.3	900	0.946	0.83
0.2	0.4	900	0.938	1.67
0.3	0.2	900	0.927	1.36
0.4	0.1	900	0.954	0
0.5	0.4	900	0.940	1.46

where the optimal α-expected efficient value is the maximum one of the five α-expected efficient values in Table 6. Generally, findings from the above tables can be summarized as follows:

Finding 1: Table 3 indicates that the predicted efficiencies of the five DMUs (gas stations) become larger as the risk criterion increase.

Finding 2: Table 4 indicates the high correlation rates have obtained for both periods (0.732 and 0.957) between predicted and real efficiency scores; it can represents the validity of the proposed DRF-DEA model (22).

Finding 3: The comparsion between predicted and real efficienies scores in Table 4 reveals significant improvement in discriminating power.

Finding 4: Table 5 indicates that our hybrid algorithm outperforms the Qin's hybrid GA algorithm in terms of runtime and solution quality.

Finding 5: It can be seen from Table 6 that the relative errors do not exceed 1.67%, which implies that the our MC-GA algorithm is robust for parameters selection and effective for solving the final proposed DRF-DEA model.

8. Conclusions

This paper attempted to present a new random fuzzy DEA model with common weights under mean chance constraints and the expected values of objective functions in dynamic environment; in which data were changing sequentially. The proposed DRF-DEA model incorporates future information on outputs and inputs into its analytical framework to predict efficiency scores of DMUs for the next financial periods. The major results of the paper include the following : (i) A new multi-objective DEA model was built in uncertain and dynamic environments, in which the outputs and inputs are characterized by random traingular fuzzy variables with normal distribution. Under this assumption, the initial proposed DRF-DEA model transformed to its equivalent multi-objecyives stochastic model; in which some constraints contain the standard normal distribution function and others are the mathematical expectation for functions of the normal random variable. (ii) In order to improve in computational time during the solution process, the equivalent multi objecyives stochastic model converted to one objective stochastic model (final repesentation of DRF-DEA model) by fuzzy multiple objectives linear programming approach. (iii) To solve the final DRF-DEA model, this paper designed a new hybrid GA algorithm by incorporating MC simulation and GA, in which the MC simulation was employed to compute the integrals which involved in some of the constraints, while GA was used to find the optimal solution of problem. To document its practicality, the DRF-DEA model was applied to predict efficiencies for five gas stations in a Iranian petroleum company for the next two financial periods. The computational results showed that our hybrid algorithm outperforms the hybrid algorithm which was proposed by Qin and Liu (2010) in terms of runtime and solution quality.

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