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An Assessment Method for Project Cash Flow under Interval-Valued Fuzzy Environment

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Abstract

Effective project management requires reliable knowledge of cash required in different stages of project life cycle. Getting this knowledge is highly dependent on sophisticated consideration of project environment. Nature of projects and their environments is associated with uncertain conditions. In this paper, a new project cash flow assessment method based on project scheduling is proposed to foresee projects' cash flow in their different stages. Interval-valued fuzzy sets (IVFSs) are applied to address the uncertainty of activity durations and costs. First, an IVF-project scheduling method is proposed to calculate early start time and early finish time of activities under IVF-environment; accordingly, a new method of cash flow assessment is introduced under IVF-environment. For the purpose of illustration, the proposed method is implemented to generate cash flow of main activities of a large-scale project. The results show the flexibility of the presented assessment method in expressing uncertainty, in addition to its capability in risk evaluation. Furthermore, using alpha-cuts to address different levels of uncertainty and risk provides a comprehensive insight into the cash required in different stages of project life cycle under different levels of risk and uncertainty. Finally, the results are discussed, and the proposed method is believed to be useful in the project evaluation.

Keywords: Cost forecasting, Project cash flow, Fuzzy project scheduling, Assessment method, Interval-valued fuzzy sets (IVFSs).

1. Introduction

Project profitability in construction industry is highly influenced by cash (Hwee and Tiong, 2002; Ebrahimnejad et al., 2012). Inability in providing for daily activities of projects caused by ineffective cash flow control could lead to projects failure (Khosrowshahi and Kaka, 2007). Therefore, the project manager is highly dependent on reliable project cash flow to foresee, analyze, and make effective decisions to deal with the potential problems. Critical path method (CPM) assumes durations of project activities to be deterministic and known, while, in reality, they are rarely known in advance (Zammori et al., 2009). Also, CPM is not capable of modeling the unpredictable nature of project activities (Barraza et al., 2000). Moreover, the standard cost flow generation derived from CPM suffers from impractical scheduling techniques. Thus, fuzzy extensions of CPM and program evaluation and review technique (PERT) have been proposed in recent decades as efforts to tackle these problems. While it is widely accepted that effective project management requires managing uncertainty, satisfying this requirement by a more sophisticated approach could be a vital step before getting practical results (Atkinson et al., 2006).

Sophisticated uncertainty management requires sophisticated tools. Most of the previous fuzzy-based approaches were based on classic fuzzy sets theory (Zadeh, 1965). In fuzzy sets theory, the decision-maker (DM) faces difficulty when expected to give an exact opinion in a number in interval [0, 1]. Expressing this degree of uncertainty by an interval is a possible solution. Interval-valued fuzzy sets (IVFSs) by Grattan-Guinness (1975) are appropriate tools for this matter. This practical extension of the fuzzy sets theory replaces traditional [0, 1] - valued membership degrees by intervals in [0, 1]. This is how the DM can express unknown and vague membership degrees. IVFSs allow for addressing the lack of information and vagueness based on feelings rather than mere facts or proof. Another advantage of the IVFS is its ease in application in comparison with type-2 fuzzy sets (Cornelis et al., 2006).

In this paper, project cash flow is generated based on IVF-project scheduling. This approach simultaneously gives the advantages of IVF-sets and fuzzy project scheduling. Fuzzy scheduling is firstly introduced to calculate early start time and early finish time of activities with IVF-numbers; then, cash flow generation approach is presented based on the IVF-scheduling. Moreover, alpha-cuts of IVF-numbers are applied in the proposed assessment method to consider different levels of

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4. Interval-valued Fuzzy Cash Flow Generation Approach

Cost per unit of time calculation is required in order to generate project cash flow. By introducing IVF-project scheduling, uncertainty in activities' durations will be considered in the cash flow. Therefore, durations differ for activities in different scenarios, and this approach leads to indifferent results for the cash flow. In order to get an insight into the longest and shortest durations, activities starting in the earliest possible time and lasting the least duration ($Min\ Duration_\alpha$) and activities starting in the latest possible time and lasting the longest duration ($Max\ Duration_\alpha$) should be considered in cash flow generation. Duration calculation for activities with IVF-time method can be extended based on (Maravas and Pantouvakis, 2010). The resulting IVF-CPM not only enables the project managers to consider uncertainty in a more practical way, but also provides a more thorough understanding of the activities durations. These durations for activities with early start $\tilde{E}S = [(es_1^U, es_1^L), es_2, (es_3^L, es_3^U)]$ and early finish $\tilde{E}F = [(ef_1^U, ef_1^L), ef_2, (ef_3^L, ef_3^U)]$ are calculated as follows:

$$Min\ D_\alpha = [min\ duration\ _\alpha^L, min\ duration\ _\alpha^U] \tag{5}$$

$$Min\, D\ _\alpha^L = [inf ES_\alpha^L, inf ES_\alpha^L + inf D_\alpha^L] = [\frac{\alpha}{\lambda}(es_2 - es_1^L) + es_1^L, \frac{\alpha}{\lambda}(ef_2 - ef_1^L) + ef_1^L] \tag{6}$$

$$Min\, D\ _\alpha^U = [inf ES_\alpha^U, inf ES_\alpha^U + inf D_\alpha^U] = [\frac{\alpha}{\rho}(es_2 - es_1^U) + es_1^U, \frac{\alpha}{\rho}(ef_2 - ef_1^U) + ef_1^U] \tag{7}$$

$$MaxD_\alpha = [max\ duration\ _\alpha^L, max\ duration\ _\alpha^U] \tag{8}$$

$$Max\, D\ _\alpha^L = [sup\ ES_\alpha^L, sup\ ES_\alpha^L + sup\ D_\alpha^L] = [\frac{\alpha}{\lambda}(es_2 - es_3^L) + es_3^L, \frac{\alpha}{\lambda}(ef_2 - ef_3^L) + ef_3^L] \tag{9}$$

$$Max\, D\ _\alpha^U = [sup\ ES_\alpha^U, sup\ ES_\alpha^U + sup\ D_\alpha^U] = [\frac{\alpha}{\rho}(es_2 - es_3^U) + es_3^U, \frac{\alpha}{\rho}(ef_2 - ef_3^U) + ef_3^U] \tag{10}$$

where $Min\, D\ _\alpha^L$ and $Min\, D\ _\alpha^U$ represent α-cuts of minimum duration in the lower and upper IVF-numbers, $Max\, D\ _\alpha^L$ and $Max\, D\ _\alpha^U$ denote α-cuts of maximum duration in the lower and upper IVF-numbers, respectively. ES_α^L and ES_α^U are α-cuts of IVF-early start of the lower and upper numbers, sup is supremum, inf is infimum, and λ and ρ are membership degrees of the lower and upper numbers, respectively. It should be noted that $0 \leq \alpha \leq \lambda$.

Fig. 1 displays an activity with early start of [(1,2), 3, (4,5)], duration of [(5,6), 7, (8,9)], and early finish of [(6,8), 10, (12,14)]. This activity has durations of $min\, D\ _0^L$= [2, 8], $max\, D\ _0^L$ = [4,12], $min\, D\ _0^U$= [1,6], and $max\, D\ _0^U$=[5,14]. Each one of the maximum and minimum durations is separately calculated for the upper and lower fuzzy numbers. Consequently, uncertainty can play a more vital role in duration calculations, and since activity cost is distributed in these intervals, uncertainty is also calculated in cash distribution. In the best possible scenario, the activity can start at interval of [1, 2] and end at [6, 8]; in the worst case, it can start at interval of [4, 5] and last till [12, 14]. All the calculations for different α-levels can be made.

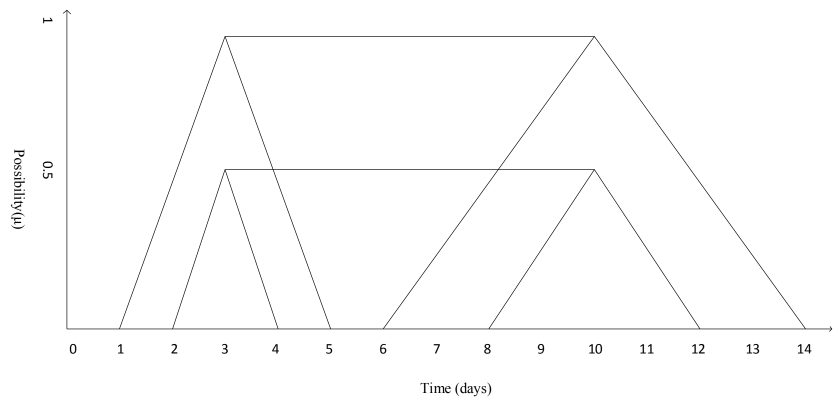

Fig. 1. IVF activity start time and completion time

$$CFU_{\alpha}^{U} = \max CFt_{\alpha}^{U} - \min CFt_{\alpha}^{U} = \sum_{t=0}^{T}\sum_{i=1}^{n} \frac{\frac{\alpha_c}{\rho_c}(ci_2 - ci_3^U) + ci_3^U}{\frac{\alpha_d}{\rho_d}(di_2 - di_1^U) + di_1^U} - \sum_{t=0}^{T}\sum_{i=1}^{n} \frac{\frac{\alpha_c}{\rho_c}(ci_2 - ci_1^U) + ci_1^U}{\frac{\alpha_d}{\rho_d}(di_2 - di_3^U) + di_3^U} \tag{25}$$

CFU_{α} displays uncertainty in working capital requirement throughout the project implementation. This measure helps the project manager to manage working requirement capitals more efficiently. The main purpose of cash flow management is to make a balance between investment capital and working capital; thus, having a comprehensive understanding of uncertainty enables the manager to achieve this goal, in addition to avoiding any unpleasant surprises.

5. Application Example

For the purpose of illustration, the proposed method is applied to generate cash flow for a network of main activities in a construction project. The activity network is displayed in Fig. 2, and the adopted data consisting of activities with IVF-durations and costs are displayed in Table 1. The membership degrees of the lower and upper numbers for all durations and costs are considered as 1 and 0.6, respectively.

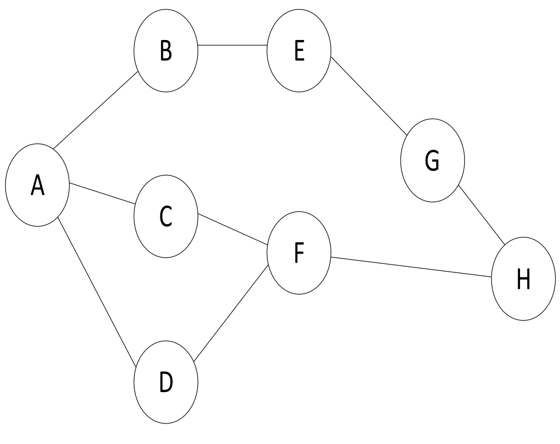

Fig. 2. Sample activity network

5.1. Computational results

IVF-project scheduling is applied to calculate early start time and early finish time activities. The corresponding results are presented in Table 2. Based on the achieved results, IVF-Gantt chart is illustrated in Fig. 3. This chart illustrates a two-dimensional Gantt schedule with the project of early start and finish dates, which are presented by IVF-numbers and calculated from the forward pass. The x-axis denotes time, while the y-axis demonstrates the activity name and the membership function which is the possibility level. Unlike conventional Gantt charts, activity durations are defined by two IVFSs which are the IVF-start date and the IVF-completion date. This approach provides the actual start and completion activity limits. Cash distributions for all activities are calculated at three different α–cuts, and the results are displayed in Table 3.

Table1
Activity network data

Activity	Predecessor	Lag	IVF-duration(days)	IVF-cost($)
A	-	0	[(5,7),8,(9,11)]	[(20,35),40,(45,60)]
B	-	0	[(2,4),6,(7,9)]	[(10,25),30,(35,50)]
C	A	0	[(16,18),20,(23,25)]	[(220,235),240,(245,260)]
D	A	2	[(10,12),15,(17,19)]	[(130,145),150,(155,170)]
E	A,B	0	[(6,8),10,(11,13)]	[(20,35),40,(45,60)]
F	C,D	0	[(7,9),10,(12,14)]	[(60,75),80,(85,100)]
G	E	0	[(6,8),10,(12,14)]	[(40,55),60,(65,80)]
H	F,G	0	[(1,3),4,(5,7)]	[(10,25),30,(35,50)]

Table 2
IVF-project scheduling

Activity	Early start	Early finish
A	[(0,0),0,(0,0)]	[(5,7),8,(9,11)]
B	[(0,0),0,(0,0)]	[(2,4),6,(7,9)]
C	[(5,7),8,(9,11)]	[(21,25),28,(32,36)]
D	[(7,9),10,(11,13)]	[(17,21),25,(28,32)]
E	[(5,7),8,(9,11)]	[(11,15),18,(20,24)]
F	[(21,25),28,(32,36)]	[(28,34),38,(44,50)]
G	[(11,15),18,(20,24)]	[(17,23),28,(32,38)]
H	[(28,34),38,(44,50)]	[(29,37),42,(49,57)]

Table 3
Cash distributions

Activity	$\alpha = 0$				$\alpha = 0.5$				$\alpha = 1$	
	Min l	*Max l*	*Min u*	*Max u*	*Min l*	*Max l*	*Min u*	*Max u*	*Min u*	*Max u*
A	3.8	6.4	1.8	12	4.8	5.2	3.1	7.7	5	5
B	3.5	8.7	1.1	25	4.72	5.4	2.6	10	5	5
C	10.2	13.6	8.8	16.25	11.6	12.2	10.2	13.8	12	12
D	8.5	12.9	6.8	16.25	9.7	10.4	8.2	12.8	10	10
E	3.1	5.6	1.5	10	3.8	4.2	2.6	6.2	4	4
F	6.25	9.4	4.2	14.2	7.6	8.2	5.8	10.6	8	8
G	4.58	8.12	2.8	13.3	5.7	6.2	4.1	8.7	6	6
H	5	11.6	1.42	50	7	8	3.6	16	7.5	7.5

uncertainty and required resources, and consequently avoided unpleasant surprises. Employing IVFSs provided the method with more flexibility in addressing uncertainty along with adding fuzzy type II advantages without inheriting its complexity. IVFSs enabled the method to be effectively applied to projects, like new product development (NPD), in which the existing information was vague and unknown. The proposed method could be useful in feasibility study, in addition to implementation stage. Infact, the results could be used as an input in project evaluation methods such as net present value and rate or return. The assessment method's application was illustrated by a practical example in construction industry. In the application example, IVF-early start time and early finish time were calculated and IVF-Gantt chart was displayed. Cash distribution for activities at different α–cuts was also presented to illustrate the impacts of different risk levels on the project cash flow. Finally, the results of method application were discussed. Applying this method as an evaluation tool in earned value analysis could be a promising research direction.

Acknowledgments

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Appendix A

In the following, some basic concepts of the IVFSs are introduced.

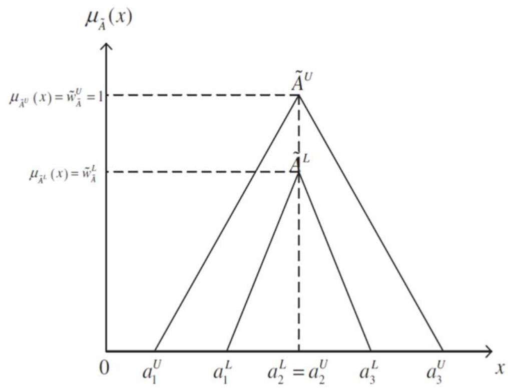

Fig. 1A. An interval-valued triangular fuzzy number $\tilde{A}$ ($\widehat{W}^L_{\tilde{A}} \neq \widehat{W}^U_{\tilde{A}}$)

A triangular interval-valued fuzzy number is shown in Fig. 1A, in which $\tilde{A}^L$ and $\tilde{A}^U$ represent the lower and upper triangular interval-valued fuzzy numbers, and $\widehat{W}^L_{\tilde{A}}$ and $\widehat{W}^U_{\tilde{A}}$ are the degrees in which event x may be a member of the lower and upper numbers, respectively (Yao and Lin, 2002), which can be described as:

$$\tilde{A} = [\tilde{A}^L_x, \tilde{A}^U_x] = [(a^L_1, a^L_2, a^L_3; w^L_{\tilde{A}}), (a^U_1, a^U_2, a^U_3; w^U_{\tilde{A}})] \quad (1A)$$

Arithmetic operations between two triangular interval-valued fuzzy numbers $\tilde{A}$ and $\tilde{B}$ displayed as $\tilde{A} = [(a^U_1, a^L_1), a_2, (a^U_3, a^L_3)]$ and $\tilde{B} = [(b^U_1, b^L_1), b_2, (b^U_3, b^L_3)]$, respectively, are as follows (Chen, 1997; Hong and Lee, 2002; Chen and Chen, 2008; Vahdani et al, 2010; Mousavi et al., 2013):

Addition of interval-valued fuzzy numbers $\oplus$:

$$\tilde{A} \oplus \tilde{B} = [(a^U_1, a^L_1), a_2, (a^U_3, a^L_3)] \oplus [(b^U_1, b^L_1), b_2, (b^U_3, b^L_3)] = [(a^U_1 + b^U_1, a^L_1 + b^L_1), a_2 + b_2, (a^L_3 + b^L_3, a^U_3 + b^U_3)]. \quad (2A)$$

Subtraction of interval-valued fuzzy numbers $\ominus$:

$$\tilde{A} \ominus \tilde{B} = [(a^U_1, a^L_1), a_2, (a^U_3, a^L_3)] \ominus [(b^U_1, b^L_1), b_2, (b^U_3, b^L_3)] = [(a^U_1 - b^U_3, a^L_1 - b^L_3), a_2 - b_2, (a^L_3 - b^L_1, a^U_3 - b^U_1)]. \quad (3A)$$

Appendix B

In the following, the ranking method of IVF-numbers introduced by Su (2007) is presented.

Let $\tilde{A} = [\tilde{A}^L_x, \tilde{A}^U_x] = [(a^L_1, a^L_2, a^L_3; w^L_{\tilde{A}}), (a^U_1, a^U_2, a^U_3; w^U_{\tilde{A}})]$, then the signed distance of $\tilde{A}$ from $\tilde{0}$ is calculated as follows:

$$(\tilde{A}, \tilde{0}) = \frac{1}{w^L_{\tilde{A}}} \int_0^{w^L_{\tilde{A}}} d^*([A^U_l(\alpha), A^L_l(\alpha)] \cup [A^L_r(\alpha), A^U_r(\alpha)], 0) d\alpha + \frac{1}{w^U_{\tilde{A}} - w^L_{\tilde{A}}} \int_{w^L_{\tilde{A}}}^{w^U_{\tilde{A}}} d^*([A^U_l(\alpha), A^L_r(\alpha)], 0) d\alpha = \frac{1}{8}[6a_2 + a^L_1 + a^L_3 + 4a^U_1 + 4a^U_3 + 3(2a_2 - a^U_1 - a^L_1) \frac{w^L_{\tilde{A}}}{w^U_{\tilde{A}}} \quad (1B)$$

Based on the aforementioned distance calculation method, two IVF-numbers will be ranked as follows:

$$\tilde{A} = [\tilde{A}^L_x, \tilde{A}^U_x] = [(a^L_1, a^L_2, a^L_3; w^L_{\tilde{A}}), (a^U_1, a^U_2, a^U_3; w^U_{\tilde{A}})]$$

$$\tilde{B} = [\tilde{B}^L_x, \tilde{B}^U_x] = [(b^L_1, b^L_2, b^L_3; w^L_{\tilde{b}}), (b^U_1, b^U_2, b^U_3; w^U_{\tilde{B}})]$$

$$\tilde{B} \leq \tilde{A} \ \ if \ \ d(\tilde{B}, \tilde{0}) < d(\tilde{A}, \tilde{0})$$

$$\tilde{B} \approx \tilde{A} \ \ if \ \ d(\tilde{B}, \tilde{0}) = d(\tilde{A}, \tilde{0})$$

References

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