

# Two-Echelon Supply Chain Considering Multiple Retailers with Price and Promotional Effort Sensitive Non-Linear Demand

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## Abstract

This study deals with the effects of a supply chain(SC) with single product, multiple retailers and a manufacturer, where the manufacturer (he) produces lot size of the product that contains a random portion of imperfect quality item. The imperfect quality products are sold in a secondary shop. The new contribution of this paper is a new non-linear demand function.

Demand of the end customers varies with pricing and promotional effort of the rivalry amongst the retailers which can be used for the electronic goods, new lunched products, etc. We investigate the behavior of the supply chain under Manufacturer-Stackelberg(MS), and Retailer-Stackelberg(RS) model structures. The nature of the mentioned models provides great insights to a firm's manager for achieving optimal strategy in a competitive marketing system. Within the framework of any bilevel decision problem, a leader's decision is influenced by the reaction of his followers. In MS model structure, following the method of replacing the lower level problem with its Kuhn-Tucker optimality condition, we transform the nonlinear bi-level programming problem into a nonlinear programming problem with the complementary slackness constraint condition. The objective of this paper is to determine the optimal selling price and promotional effort of each retailer, while the optimal wholesale price of the perfect quality products are determined by the manufacturer so that the above strategies are maximized. Finally, numerical examples with sensitivity analysis of the key parameters are illustrated to investigate the proposed model.

**Keywords:** Supply chain(SC), Game theory, Promotional effort, Pricing, Kuhn-Tucker optimality condition

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## 1. Introduction

In the definition of SC it is possible to find a number of issues such as price, promotional effort, quality of products, coordination, etc. Studies regarding its effects on supply chain management still remain sparse to our knowledge. This paper considers a SC through pricing and promotional effort strategies.

One of the major issues in supply chain is to meet customer requirements to avoid losing customers' good will. The customers are more interested for the products which have better quality in a reasonable price. To improve customer satisfaction, the good management has given emphasize on the aspect of quality in supply chain management, for instance, the percentage of good quality products in any manufacturing system could not reach at 100%. Consequently, it is necessary to separate imperfect quality products from the whole lot by screening process to sustain the existence of a business sector in a competitive market. Another major challenges in supply chain is setting sales prices. Price of product has a direct effect on the consumers' demand. In this era, pricing strategies of commodities play an important role in capturing the market. Besides pricing strategies, promotion of sale of products is a form of marketing, advertising, etc.

Promotion for sales acts as an important tool to persuade and inform customers about the products and services they have to offer. The basic philosophy of consumers is that they want the best quality of products at lowest possible price, as they have an open choice to select the products in an oligopoly system.

Many researchers have concluded that centralized supply chain has a better result. But in most supply chains, the supply chain members have conflicting goals and there is no central decision-maker who can control all supply chain members. Manufacturer and the retailer often make their own decision. They try to optimize their decision by considering the other player's decision. The manufacturer makes pricing decision by considering the retailer's order quantity. And when the retailer decides the optimal ordering quantity, he will set the wholesale price. This situation has typical characteristics of the Stackelberg game. A standard Stackelberg game involves two players: a leader and a follower. The leader takes actions first and then the follower reacts to the leader's decisions rationally. Therefore, the Stackelberg game is a sequential game. Mathematically, a Stackelberg game can be formulated as a bi-level programming problem (Colson et al., 2007).

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Now, our aim is to develop a supply chain model taking into account the important factors such as price, promotion. First, we will review the existing literature related to the study.

**2. Literature Review**

Optimal pricing strategy (Panda et al., 2013; Panda and Saha, 2010; Shah and Raykundaliya, 2010; Salvietti et al., 2014 ) is a major issue to attract the customers in any business organization in a given economy. Coordinating pricing decision in supply chain under different channel structure has been extensively studied in the marketing and operation management literature.

Tsao and Sheen, (2012) studied two-echelon with multiple-retailer distribution channel under retailers’ promotional efforts and the sales learning curve. They considered both situations where retailers bear the promotion cost and the supplier shares the promotion cost. The first Stackelberg game was described by the German economist Heinrich Freiherr von Stackelberg in 1934, who studied the competition between two firms selling a homogeneous good (Von Stackelberg et al., 2010). The concept of Stackelberg game was then extended to a variety of research fields and applications to study the situation where a leader–follower relationship is observed (Chu and You, 2014; Chu et al., 2015).

In this paper, we investigate a decentralized two-echelon supply chain model with price, and promotional effort sensitive demand where the manufacturer produces both conforming and nonconforming products and the nonconforming products are sold in a secondary shop at lower price.

We discuss the model under the MS, and RS model structure. We optimize all the above-mentioned structures with respect to the decision parameters. The remainder of the paper is organized as follows. Fundamental assumptions and notations are provided in Section 3. Formulation of the model is discussed in Section 4. Section 5 provides the numerical analysis. Sensitivity analysis is demonstrated in Section 6. Finally, conclusions to the paper are drawn in Section 7.

**3. Fundamental Assumptions and Notations**

*3.1. Assumptions*

The following assumptions are made to develop the model:

- (i) The model is developed for a single product with a single manufacturer and multiple retailers.
- (ii) The production system of the manufacturer produces both perfect and imperfect quality products.
- (iii) The manufacturer performs screening process to separate perfect and imperfect quality products, After receiving the lot. Manufacturer supplies perfect quality product to multiple retailers to satisfy their demand.
- (iv) The production rate of imperfect quality products is a random variable and follows a probability distribution. These items are sold in a single lot by others, after completion of the whole production.

- (v) The production rate of perfect quality products by the manufacturer is equal to the demand rate of the customers.
- (vi) The demand rate of the customers depends on selling price and promotional effort.

*3.2. Notations*

The following notations are used throughout the paper:

Table 1  
Parameters and Decision Variables

$D_m$	Demand of manufacture;
$D_i^r$	Demand of $i$ th ( $i = 1, 2, \dots, n$ ) retailer;
$\rho_i$	Promotional effort of $i$ th ( $i = 1, 2, \dots, n$ ) retailer;
$p_i$	Unit selling price of $i$ th ( $i = 1, 2, \dots, n$ ) retailer;
$w_{mp}$	Unit wholesale price of perfect product determined by manufacture
$w_{ml}$	Unit wholesale price of imperfect product determined by manufacture
$c_m$	Manufacture's unit purchasing cost;
$c_i^r$	$i$ th ( $i = 1, 2, \dots, n$ ) retailer's unit purchasing cost;
$\pi_{ri}$	Profit function of $i$ th ( $i = 1, 2, \dots, n$ ) retailer;
$\pi_m$	Profit function of manufacture
$s$	Cost of screening per unit product;
$\alpha$	Proportional probability of imperfect items, a random variable;
$f(\alpha)$	The probability density function of the random variable $\alpha$ ;
$m, r$	Denotes the manufacture and retailer, respectively;

Superscripts (\*) denotes optimal value.

**4. Formulation of the Model**

In the paper, we develop a two-echelon supply chain model where the manufacturer produces items for retailers, and retailers sell those to the customers. The production system is not perfect. The system produces imperfect quality products at a random proportion  $\alpha$  ( $0 \leq \alpha \leq 1$ ) imperfect quality product that follows a probability distribution. After producing the lot, manufacturer performs screening process to separate perfect and imperfect quality products. manufacturer supplies perfect quality product to retailers to satisfy their demand. Therefore, the production rate of the manufacturer is  $(1 + E(\alpha))$  times normalized market demand. The manufacturer sells the imperfect quality products in a lot to other markets or secondary shops. The rate of demand of competitive retailers depends on competitive retail price. In  $i$ th retailer faces a price sensitive non-linear demand  $d_i^r(p) = A_i p_i^{-e_i} \prod_{j=1, j \neq i}^n p_j^\gamma$  (as

like Choi, 1991), where  $p_i$  denotes the price charged by the  $i$ th retailer and the parameters are all positive real constants. It is required that

$e_i > (n - 1)\gamma$ , for  $i = 1, 2, \dots, n$  for the value of the parameters as it is reasonable that sales items are relatively more sensitive to price at that retailer than prices at the competence retailers. Observing the market demand, each retailer provides promotional effort to enhance the demand of their product. Here, the  $i$ th retailer's promotional effort  $\rho_i$  increases the market demand  $D_i^r(\mathbf{p}) = \rho_i d_i^r(\mathbf{p})$  with the cost of effort  $C_r(\rho_i) = k_i d_i^r(\mathbf{p})(\rho_i - 1)^2$  which is convex, increasing and continuously differentiable function of the promotional effort ( $\rho_i$ ), for any  $\rho_i \geq 1$  (like Krishnan et al., 2004).

Now, the profit of the manufacturer is

$\pi_m$  = Sales revenue of perfect quality products - Production cost of products -

Screening cost of products + Sales revenue of the imperfect quality products

$$= (w_{mp} + E(\alpha)w_{ml} - (1 + E(\alpha))(s + c_m)) \left( \sum_{i=1}^n D_i^r \right) \quad (1)$$

The profit of  $i$ th retailer is given as

$\pi_i^r$  = Sales revenue of perfect products - Purchasing cost of perfect quality products - Promotional effort cost

$$= (p_i - w_{mp} - c_i^r) D_i^r - k_i d_i^r(\mathbf{p})(\rho_i - 1)^2 \quad (2)$$

$$s. t. \quad p_i > w_{mp} + c_i^r \quad (3)$$

#### 4.1. MS model

In this case, the manufacturer as a leader makes the decision on wholesale price of perfect product after seeing the reaction of  $i$  thretailer on observing price and promotional effort. Here,  $i$  thretailer first calculates the selling price ( $p_i$ ) and the promotional effort ( $\rho_i$ ) for a given wholesale price of perfect product ( $w_{mp}$ ) of the manufacturer.

Mathematically, a single-leader-multiple-followers Stackelberg game can be formulated as a bilevel programming problem (BP) (Colson et al., 2007), Then the BP problem is transformed as

$$\begin{aligned} p: \quad & \max_{w_{dp} \geq 0} \pi_m \\ s. t. \quad & \max_{p_i, \rho_i \geq 0} \pi_i^r, \quad i = 1, \dots, n \\ & s. t. \quad p_i > w_{mp} + c_i^r \end{aligned} \quad (4)$$

This paper transforms the nonlinear BP to the differentiable nonlinear programming problem equivalently in the sense of a global optimal solution and then propose a simple algorithm for the nonlinear BP. To obtain optimal decision variables, we firstly formulate the

penalty problem of (4), and adopt the method of replacing the lower level problem with its Kuhn-Tucker optimality condition. Then we append the complementary slackness condition to the upper level objective with a penalty.

##### 4.1.1. Best responses of retailers

Suppose that the strategies of manufacture are fixed. According to Ref. (Pal et al., 2015), the concavity of each retailer's profit function was proved with respect to their decision variables.

Also constraints of each retailer is linear, so constraint set of each retailer is convex. then following (Lv et al., 2007), the bilevel programming (1) exists optimal solutions. Thus, through the Karush-Kuhn-Tucker (KKT) condition, the Lagrange equation and first order condition of each retailer's objective function are as follows (Equations (5) to (10)).

$$\begin{aligned} L_i^r(p_i, \rho_i, \lambda_i) &= \pi_i^r(p_i, \rho_i) + \lambda_i (p_i - w_{mp} - c_i^r) \\ &= (p_i - w_{mp} - c_i^r) D_i^r \\ &\quad - k_i d_i^r(\mathbf{p})(\rho_i - 1)^2 + \lambda_i (p_i - w_{mp} - c_i^r) \end{aligned} \quad (5)$$

$$\nabla L_i^r(p_i, \rho_i, \lambda_i) = 0 \rightarrow \nabla \pi_i^r(p_i, \rho_i) + \lambda_i \nabla (p_i - w_{mp} - c_i^r) = 0 \quad (6)$$

$$\begin{aligned} \left[ \frac{\partial \pi_i^r(p_i, \rho_i)}{\partial p_i} \right] \left[ \frac{\partial (p_i - w_{mp} - c_i^r)}{\partial p_i} \right] \\ + \lambda_i \left[ \frac{\partial \pi_i^r(p_i, \rho_i)}{\partial \rho_i} \right] \left[ \frac{\partial (p_i - w_{mp} - c_i^r)}{\partial \rho_i} \right] \leq 0 \end{aligned} \quad (7)$$

$$\lambda_i \cdot (p_i - w_{mp} - c_i^r) = 0 \quad (8)$$

$$-p_i + w_{mp} + c_i^r \leq 0 \quad (9)$$

Obtaining a closed-form analytical solution for  $p_i$ ,  $\rho_i$  is less possible, but paying attention to these relationships is especially valuable in the numerical analysis. We adopt the method of replacing the lower level problem with its Kuhn-Tucker optimality conditions and reduce (4) to the following nonlinear programming problem

$$\begin{aligned} p: \quad & \max_{p_i, \rho_i, w_{dp} \geq 0} (w_{mp} + E(\alpha)w_{ml} \\ & \quad - (1 + E(\alpha))(s + c_m)) \left( \sum_{i=1}^n D_i^r \right) \\ & \quad \lambda_i^T \cdot (p_i - w_{mp} - c_i^r) = 0 \\ & \quad \left[ \begin{aligned} e_i k_i p_i^{-1-e_i} \prod_{(j=1)(j \neq i)}^n p_j^\gamma (\rho_i - 1)^2 + p_i^{-e_i} \rho_i \prod_{(j=1)(j \neq i)}^n p_j^\gamma - (1) \\ - 2k_i p_i^{-e_i} (-1 + \rho_i) \prod_{(j=1)(j \neq i)}^n p_j^\gamma + p_i^{-e_i} \end{aligned} \right. \quad (1) \\ & \quad \leq 0 \\ & \quad p_i - w_{mp} - c_i^r \geq 0 \end{aligned}$$

$$\lambda_i^T \cdot (p_i - w_{mp} - c_i^r) = 0$$

For problem (10), due to the existence of the complementary slackness condition, similar to Ref. (Pan et al., 2010), we append the complementary slackness

condition to the upper level objective with a penalty and obtain the following penalized problem:

$$\begin{aligned}
 (M): \quad & \max_{p_i, \rho_i, w_{mp} \geq 0} [(w_{mp} + E(\alpha)w_{ml} - (1 + E(\alpha))(s + c_m)) \left( \sum_{i=1}^n D_i^r \right) - M \sum_{i=1}^n \lambda_i^T (p_i - w_{mp} - c_i^r)] \\
 & \text{s.t.} \\
 & p_i - w_{mp} - c_i^r \geq 0, \quad i = 1, \dots, n \\
 & \lambda_i + e_i k_i p_i^{-1-e_i} \prod_{(j=1)(j \neq i)}^n p_j^\gamma (\rho_i - 1)^2 + p_i^{-e_i} \rho_i \prod_{(j=1)(j \neq i)}^n p_j^\gamma \\
 & \quad - e_i p_i^{-1-e_i} (-c_i^r + p_i - w_{mp}) \rho_i \prod_{(j=1)(j \neq i)}^n p_j^\gamma \leq 0, \quad i = 1, \dots, n \\
 & -2k_i p_i^{-e_i} (-1 + \rho_i) \prod_{(j=1)(j \neq i)}^n p_j^\gamma + p_i^{-e_i} (-c_i^r + p_i - w_{mp}) \prod_{(j=1)(j \neq i)}^n p_j^\gamma \leq 0, \\
 & p_i, \rho_i, w_{dp}, \lambda_i \geq 0, \quad i = 1, \dots, n
 \end{aligned} \tag{11}$$

**RS model**

In this case, both retailers as leaders make decisions simultaneously on prices and promotional efforts after seeing the reaction of the manufacturer on observing wholesale price of perfect product. Based on the reaction of the manufacturer, retailers optimizes their profits with respect to the selling price ( $p_i$ ) and the promotional effort( $\rho_i$ ).

$$\begin{aligned}
 \mu_{r_i} > \mu_m & \rightarrow (p_i - w_{mp} - c_i^r) > w_{mp} - (s + c_m) \\
 & \rightarrow w_{mp} < \frac{p_i + (s + c_m) - c_i^r}{2}
 \end{aligned} \tag{12}$$

**4.1.2. Best responses of manufacturer**

Where  $\mu_{r_i} = p_i - w_{mp} - c_i^r$  and  $\mu_m$  is equal to  $w_{mp} - (s + c_m)$  are retailer's and manufacturer's unit margins, respectively. We have the value  $w_{mp} = w_{mp}^{RS^*}(p_i)$ , Thus, the optimal value of  $w_{mp}^{RS^*}(p_i)$  is  $\frac{p_i + (s + c_m) - c_i^r}{2}$ . Now, putting the value of  $w_{mp}^{RS^*}(p_i)$ , in the profit function (2) of each retailer, we have

Suppose that the strategies of retailers are fixed. The manufacturer calculates wholesale price of perfect product ( $w_{mp}$ ) for given  $p_i$  and  $\rho_i$ .

$$\text{s.t. } p_i > s + c_m + c_i^r, \quad \text{and } \rho_i \geq 0, \quad i = 1, \dots, n \tag{13}$$

$\pi_m$  is increasing in line with  $w_{mp}$ , which means the optimal value for  $w_{mp}$ , is  $p_i - c_i^r$ ; However,

$p_i - c_i^r > w_{mp}$ ;  $w_{mp}$  cannot be equal to  $p_i - c_i^r$ , or in other words there would be no profit for both sides.

**Proposition 4.2.1** . The profit function of each retailer for the Stackelberg retailer scenario is concave respect to  $p_i$  and  $\rho_i$ .

**Proof:** The proof is similar to Ref. (Pal et al., 2015). Also constraints of each retailer are linear, so constraint set of each retailer is convex. To obtain optimal decision variables, we use KKT conditions. Thereby, Lagrange equation and first order condition of each retailer's objective functionary as follows (Equations (14) to (18)).

We use a similar approach as previously suggested in Xie and Neyret, (2009) to handle the problem; we assume that each retailer will not sell the product if she does not obtain a minimum unit margin. We take manufacturer's unit margin level and replace the wholesale price constraint with:

$$\begin{aligned}
 L_i^r(p_i, \rho_i, \lambda_i) &= \pi_i^r(p_i, \rho_i) + \lambda_i (p_i - w_{mp} - c_i^r) \\
 &= (p_i - w_{mp} - c_i^r) D_i^r - k_i d_i^r(\mathbf{p})(\rho_i - 1)^2 + \lambda_i (p_i - s - c_m - c_i^r)
 \end{aligned} \tag{14}$$

$$\nabla L_i^r(p_i, \rho_i, \lambda_i) = 0 \rightarrow \nabla \pi_i^r(p_i, \rho_i) + \lambda_i \nabla (p_i - s - c_m - c_i^r) = 0 \tag{15}$$

$$\frac{\partial \pi_i^r(p_i, \rho_i)}{\partial p_i} + \lambda_i \frac{\partial (p_i - s - c_m - c_i^r)}{\partial p_i} \leq 0 \tag{16}$$

$$\left[ e_i k_i p_i^{-1-e_i} \prod_{(j=1)(j \neq i)}^n p_j^y (\rho_i - 1)^2 + p_i^{-e_i} \rho_i \prod_{(j=1)(j \neq i)}^n p_j^y - e_i p_i^{-1-e_i} (-c_i^r + p_i - s - c_m) \rho_i \prod_{(j=1)(j \neq i)}^n p_j^y + \lambda_i \right. \\ \left. - 2k_i p_i^{-e_i} (-1 + \rho_i) \prod_{(j=1)(j \neq i)}^n p_j^y + p_i^{-e_i} (-c_i^r + p_i - s - c_m) \prod_{(j=1)(j \neq i)}^n p_j^y \right] = 0 \tag{17}$$

$$\lambda_i \cdot (p_i - s - c_m - c_i^r) = 0 \tag{18}$$

Solving  $\frac{\partial L_i^r}{\partial p_i} = 0, \frac{\partial L_i^r}{\partial \rho_i} = 0$  for  $i=1, \dots, n$  gives:

$$\frac{\partial L_i^r}{\partial p_i} = 0 \rightarrow \lambda_i + e_i k_i p_i^{-1-e_i} (\rho_i - 1)^2 \prod_{(j=1)(j \neq i)}^n p_j^y + p_i^{-e_i} \rho_i \prod_{(j=1)(j \neq i)}^n p_j^y \\ - e_i p_i^{-1-e_i} (-c_i^r + p_i - s - c_m) \rho_i \prod_{(j=1)(j \neq i)}^n p_j^y \\ = 0 \tag{19}$$

$$\frac{\partial L_i^r}{\partial \rho_i} = 0 \rightarrow -2k_i (-1 + \rho_i) + (-c_i^r + p_i - s - c_m) = 0 \tag{20}$$

Also, optimal  $\lambda_i$  according to Equation (18), is as follows:

$$\lambda_i = \max \left\{ 0, -e_i k_i p_i^{-1-e_i} (\rho_i - 1)^2 \prod_{(j=1)(j \neq i)}^n p_j^y - p_i^{-e_i} \rho_i \prod_{(j=1)(j \neq i)}^n p_j^y \right. \\ \left. + e_i p_i^{-1-e_i} (-c_i^r + p_i - s - c_m) \rho_i \prod_{(j=1)(j \neq i)}^n p_j^y \right\} \tag{21}$$

Again obtaining a closed-form analytical solution for Eq. (20) is less possible, but paying attention to these relationships is especially valuable in the numerical analysis that we are going to make about the models. Similar to (Esmaili and Zeepongsekul, 2010; Gao et al., 2015), to solve Eq. (19), the Simpson Quadrature method could be initially used to find the value of the decision variable. Then, the problem changes to an equations system for our example, which will provide the Stackelberg retailer solution.

### 5. Numerical Example

In this section, a numerical study is carried out to demonstrate the behavior of the proposed models and to gain some insights of the problem which are respectively solved by Lingo and Matlab software and obtain the following values for  $p_i, \rho_i, wdp$ . The

experiments are implemented in the following manner. First, for all parameters of the models, we extract randomly a value out of its given interval. We extract randomly more than 100 groups of values of the parameters in total in the experiment. Then we calculate the equilibrium solution of two models based on this group of extracted values of all parameters. Our remarks below are obtained based on the computational results of all groups. We pick arbitrarily one from all groups, respectively to illustrate our observations intuitively. Suppose the random amount of imperfect quality item,  $\alpha$ , is uniformly distributed between 0.03 to 0.30. Hence,  $E(\alpha) = (0.03+0.30)/2 = 0.16$ . Optimal prices and optimal promotional efforts and individual profits and total channel profit in stackelberg games are presented in Table 2.

Table 2. Numerical data and solution for the manufacture stackelberg and retailer stackelberg game

I	MS							RS						
	$p_i$	$\rho_i$	$c_i$	$wdp$	$\pi_i$	$\pi_m$	$\pi_c$	$p_i$	$\rho_i$	$c_i$	$wdp$	$\pi_i$	$\pi_m$	$\pi_c$
1	3.60	1	3.04	1.5	72.5	7.91	7404.9	3171	29471	69.32	7.08	3444	8272	19315
2	3.67	1	3.02	1.5	100	11.92	11881			96.9	11.01	4010		
3	3.90	1	2.94	1.5	72.87	7.21	70136			70.02	6.89	3589		

We see from Table 2 that the wholesale price is comparatively lower, whereas the retailer's selling price and promotional effort are high in the MS model compared to the wholesale price, selling price, and promotional effort in the RS model. The manufacturer's

profit is less in the MS model than the RS model because the profit margin and demand are both low in the MS model. We also observe that the profit of each retailer in the RS model structure is less than that from the profit of each retailer in the MS model. In the RS model structure,

each retailer monitors the manufacturer's strategies first and then she adjusts with her own strategies. Therefore, the manufacturer sets the wholesale price  $w_{mp}$  first for given selling price  $p_i$  and promotional effort  $\rho_i$ . Now,  $i$  retailer optimizes her profit with respect to  $p_i$  and  $\rho_i$  by addressing the reaction of the manufacturer on  $w_{mp} = w_{mp}(p_i, \rho_i)$ . The price is more sensitive than promotional effort in our demand function, and the demand rate decreases strictly with increasing selling price.

As a result, each retailer's profit may not be satisfactory in the RS model. The wholesale price is high but each retailer's selling price and promotional effort are lower in the RS model compared to the wholesale price, selling price, and promotional effort in the MS model. Consequently, the profit of the retailer in the RS model structure is less than his profit in the MS model structure

Table 3  
Sensitivity of individual profits of manufacturer and retailer in different models with changes in key parameters

Parameter values	Manufacturer's profit		Retailer's profit	
	$\pi_m^{MS}$	$\pi_m^{RS}$	$\pi_r^{MS}$	$\pi_r^{RS}$
<b>e</b>				
2.5	3569.40	6578.28	7082.3	3878.6
2.75	1690.9	2844.78	3006.4	1777.5
3.25	387.9	580.2	601.4	393.13
3.5	188.08	270.07	278.42	188.3
<b>A</b>				
500000	403.322	634.826	663.1	414.97
750000	604.88	952.3	994.05	622.42
1250000	1008.21	1587.06	1657.7	1037.4
1500000	1209.97	1840.6	1989	1244

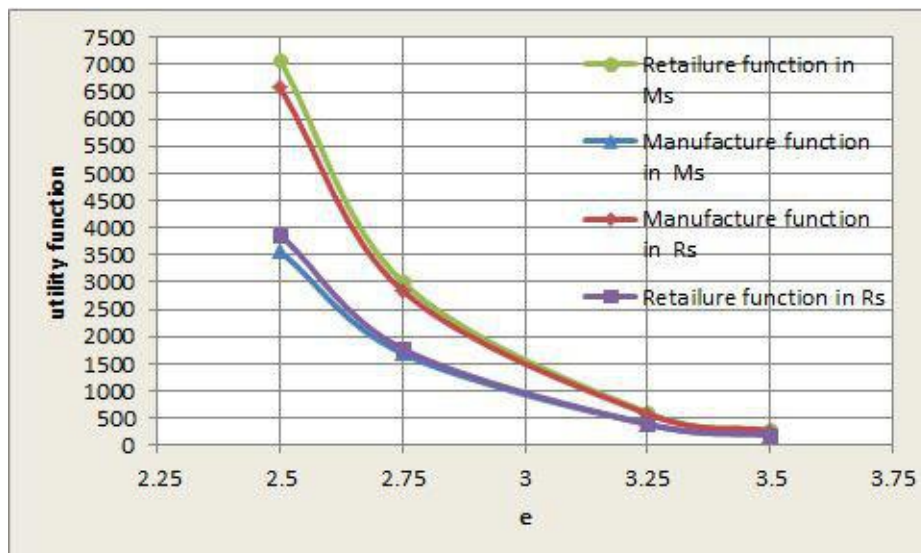


Fig.1. Sensitivity of individual profits of manufacturer and retailer in different Models with changes in parameter e

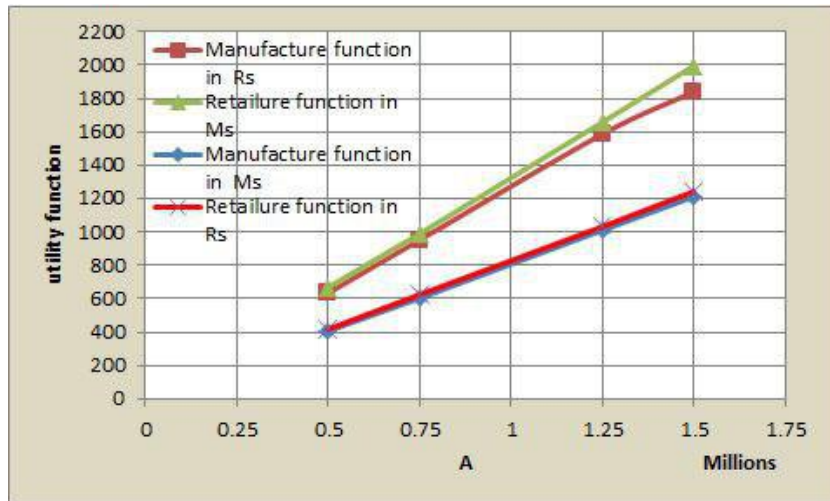


Fig.2. Sensitivity of individual profits of manufacturer and retailer in different models with changes in parameter A

## 6. Sensitive Analysis

We study the changes of optimal profit of the manufacturer and retailer (Table 3) with changes of the key parameters. The sensitivity analysis of the parameters helps the decision makers to take appropriate decisions on their marketing strategy. The following features and managerial insights are observed:

- With the increasing value of price-sensitive parameter  $e$ , the profits of the manufacturer, retailer, and total supply chain of all the model structures decrease rigorously.
- With the increasing value of scale parameter  $A$  of the demand function, optimal profits of the manufacturer, retailer, and total chain increase in all the model structures.

## 7. Conclusion

Marketing strategies, according to the capabilities of the strategies, are the most vital decisions to sustain the existence of industries in a competitive market. The companies should observe carefully the behavior of the market every day and should maintain good professional and tactical relationship with their partners. Nowadays, industrialists face great challenges to adjust their strategies on the marketing factors such as price, promotional effort, etc.

Considering the above factors, we develop and analyze the model. In the proposed model, we formulate a two-echelon single-manufacturer–multiple-retailer supply chain model with price and promotional effort sensitive demand. We study the behavior of the model in decentralized structures. We derive the optimal strategies on selling price, and promotional effort (Table 2) in decentralized structures. The optimal strategies on wholesale price of the manufacturer and the optimal strategies on selling price and promotional effort of each retailer are also computed for MS and RS decentralized structures. In the examples, (Table 2) we see that each retailer is more profitable in the MS model compared to the RS model, and the manufacturer is more profitable in the RS model compared to the MS model. The total supply chain is more profitable in MS structure than the RS structure.

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