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The Optimal Number of Hospital Beds Under Uncertainty: A Costs Management Approach

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Abstract

Equipping hospital beds uses a great deal of a hospital's resources. Therefore, it is essential to consider the hospital beds' efficiency. To increase its efficiency, a fuzzy unrestricted model for managing hospital expenses is presented in this paper. The lack of beds in hospitals leads to patients' admission loss and consecutively profit loss. On the other hand, increasing the bed count leads to an increase in equipment expenses. Therefore, in order to determine optimal bed capacity, it is of utmost importance to consider these two costs simultaneously. In our paper, hospital admission system is modeled with a multi-server queuing system (M/M/K). Therefore, to calculate the total cost function, limiting probabilities of multi-server queueing model is used. Furthermore, due to uncertain nature of parameters, such as interest rate and hospitalization profit in various future time periods, these uncertainties are covered by fuzzy logic. Finally, to determine the optimal bed count, Lee and Li's fuzzy ranking method is used. This model is implemented on a case study. Its goal is to determine the optimal bed count for emergency unit of Razi hospital in Torbat Heydarieh. Considering the high capability of Markovian chains in modeling different circumstances and the various queueing models, the proposed model can be extended for various hospital units.

Keywords : Optimal number of hospital beds, Costs management, queuing theory, Fuzzy ranking techniques.

1. Introduction

Hospital beds require large amount of investment of a hospital's resources. Therefore, managing hospital capacity and appropriate productivity are vitally important. Due to some disagreement on the effectiveness of hospital beds in Iran, the study and research is defined by the Health Ministry. Although it began in 1995, in the absence of an appropriate method of evaluation and measurement of the effectiveness of hospital beds, this project is needed to develop a new idea to determine the optimal number of hospital beds.

Incorrect and inefficient management of hospitals has led to a lot of economic pressure. Furthermore, it may cause a waste of resources, an increase in patient lists, or canceled surgeries, etc. In recent decades, the growth of the population has pushed up the demand for hospital services (Coile & Association, 2002). Many hospitals do not have enough resources and space. This causes an increase in the costs of the development of hospitals in the future (Coile & Association, 2002). Increased reception of patients and lack of hospitals lead to increased capacity of hospitals. By increasing the number of patients and the length of time, a patient is hospitalized and the need for more capacity increases, and vice versa. This issue causes challenge in determining optimal capacity (Bazzoli et al., 2003).

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2. Literature Review

Before studying recent studies on determining the optimal capacity of a hospital, it is necessary to note that some researchers have studied the capacity management issue under the name of demand management and some have used demand and capacity management. However, the issue of demand management and capacity management must be considered to be two different concepts.

Demand management tries to bring the patients' or customers' demand timing and patients or customers' demand volume under control of an organization management team, with the implementation of marketing strategies (Taylor, 1980). Capacity management, however, guarantees that sufficient capacity to meet and supply the market demand exists (Klassen & Rohleder, 2002).

Capacity-related decision making is one of the most important strategic concerns for industry's managers, and it affects the way an industry responds to current and future demands (Getz, 1983). In the capacity evaluation field, most of the previous studies have had a particular approach of their own to capacity management in different industries.

For example, most of the researchers in tourism industry have focused on the issue of capacity management using a revenue management approach (Crandall & Markland, 1996). However, the literature review on this field shows a shortage of comprehensive analysis in the capacity management field for different industries (Pullman & Rodgers, 2010). It is notable that there is more research in the service-related fields in comparison to the other fields. This is because in most service-related fields, the service capacity has prominent effect on customer satisfaction and level of service offered to them. Consequently, optimal capacity determination has been proposed in areas such as hospitals, parking areas, restaurants, hotels, and residential centers.

Furthermore, research into the determination of optimal capacity has been done in the manufacturing sector. The models that are most widely known correspond to warehousing.

For example, (White & Francis, 1971) studied the optimal warehouse capacity problem with both the deterministic and probabilistic demand conditions and proposed a solution method based on the linear programming, duality theory, and network flow. They also considered the warehouse construction costs, transportation costs, and storage costs in a public warehouse.

In the restaurant management domain, the research of Hwang and colleagues (Hwang et al., 2010) must be noted. Using queuing models, they modeled a system for a local restaurant. The model was designed in order to maximize profit and increase customer satisfaction.

A study was also done to determine the optimal capacity of hotel. By using the inventory model, (Gu, 2003) tried to analyze and optimize the capacity of a hotel and casino in Las Vegas. He proved both of the properties of single-period model (i.e., a perishable item and the probabilistic demand used by the hotel industry), and used this for the optimization of the capacity of hotels. According to his statement, if the empty and ready rooms are assumed as a product of hotel, then the rooms that do not accept the customers are the perishable products.

In the hospital issue, due to the high cost of preparing and equipping the operating rooms, many studies have been performed to determine the optimum number of operating rooms. Different approaches are presented in this study to determine the optimal capacity that can be simulated by discrete and stochastic simulation methods, as well as a combination of queuing theory or other methods (Hershey et al., 1981; Kao & Tung, 1981; Kokangul, 2008; McManus et al., 2004; Milne & Whitty, 1995).

For example, (Kokangul, 2008) proposed the combination of deterministic and stochastic models for optimizing the capacity of single hospital beds, for which the number of admissions and hospitalization length of each patient is a random and unpredictable process.

The key parameters are presented according to the level of service, the level of employment, and the rate of admissions. By using data obtained through simulation, a non-linear mathematical model is presented to determine the optimal capacity.

(Ben Bachouch et al., 2012) used linear programming for hospital bed planning. In this research, two types of patients were considered: elective and acute cases. They took into account several constraints: incompatibility between pathologies, no mixed-sex rooms, continuity of care, etc.

(Lapierre et al., 1999) used a mathematical model to develop time series models by using census data on an hourly basis (number of patients in a care unit at a given time) in order to calculate the size of each unit. This study used simulation in order to minimize the amount of collected data. By representing the time census as a time series, models produce a frequency distribution to show the bed demand.

(Walczak et al., 2003) used neural networks to predict the length of the stay of a patient arrived at emergency in order to optimize bed planning.

(Garg et al., 2010) developed a non-homogeneous discrete time Markov chain with time-dependent covariates in order to model the patient flow in a cost- or capacity-constrained healthcare system.

In another research, (Gong et al., 2010) used a multi-objective learning particle swarm optimization for the bed allocation problem in hospitals in general.

(D. K. Lee & Zenios, 2009) developed a semi-closed migration network to illustrate patient flow into the clinic and to illustrate the flow between the clinic and the hospital.

(L. Li & Benton, 2003) used structural equation modeling, taking into account several static factors such as hospital size and hospital location in order to address the hospital capacity management issue. Several managerial insights emerged from the research:

- Hospital cost control is affected by technology and workforce;
- The hospital's quality relates to the quality of the equipment and workforce chosen;
- Staff training, staff competence, and career growth influence the decisions of management;
- Location, size and service mix have significant impact on hospital's capacity related decisions.

Some studies used queuing models for solving and optimizing the industry and the healthcare problem.

(Cochran & Roche, 2009) increased the capacity of an emergency department based on a queuing model. In this model, non-homogeneous arrival patterns, non-exponential service time distributions, and multiple patient types are assumed.

In another study, (X. Li et al., 2008) used a multi-objective decision model based on queuing theory and goal programming was developed for the allocation of beds in a hospital.

In another research, by integrating queuing theory and compartmental models of flow, (Gorunescu et al., 2002) tried to demonstrate how changing admission rates, length of stay and bed allocation influence bed occupancy, emptiness and rejection in departments of geriatric medicine.

A review of capacity management studies indicated that in this area, the studies are very poor. The limited research available is used for specific conditions and the slightest changes in assumptions lose the theories' effectiveness. Therefore, in this study, an unconstrained model is

provided, using queue theory to determine the optimal capacity of the hospital.
In the proposed approach, there are several contributions which make the study unique and distinct from other studies. These contributions lead to superior results in hospital capacity management domain. The highlights of our contributions are:
• Unrestricted fuzzy non-linear model. This model is used in addition to fuzzy ranking. Together, this approach reduces execution time. Furthermore, compared to the corresponding restricted fuzzy non-linear model, this approach is understood more easily by humans.
• The use of Markovian chains and queueing theory. The high flexibility of Markov chains in modeling would lead to a suitable model for various hospitals and conditions. Therefore, the resulting model will have a high level of flexibility.
• Integration of fuzzy logic and queuing theory. This integration leads to the coverage of uncertain conditions which is an inherent condition of servicing environments, especially hospitals.
• A novel cost function with the consideration of time value of money.

3. The Proposed Model

In this section, the proposed queuing model is presented to determine optimal hospital capacity. Since the proposed model is based on queuing model concepts, it is necessary to adapt the hospital admission system to a queuing system.
In this adaptation:

- Hospital patients are customers in the queuing system with an arrival rate of λ
- Hospital beds are servers in this queuing system, and thus the number of the hospital beds is the same as that of the servers of the queue system.
- The average duration of patients' hospitalization time in hospital equals the inverse of the expected service time of the queuing system and is indicated by $\frac{1}{\mu}$.
- The average number of occupied beds in a hospital equals the expected number of customers in the queuing model which is represented by L.

3.1. Model assumptions and symbols

- The time between the patients' arrivals and the duration of patients' hospitalization time in the hospital follows an exponential distribution.

k: The capacity of the hospital (i.e., the number of the beds in hospital).

π_n: The probability of n patients hospitalized in the hospital in a long period (i.e., the percentage of the time in which the hospital has *n* occupied beds).

λ : The rate of patients' arrivals to hospital.

μ : The service rate to patients in the hospital.

P: The net profit of patients' hospitalization in each bed of hospital for one night.

b: The necessary budget for equipping and increasing the capacity by a single bed for the hospital (i.e. the cost of allocating and building the necessary space for a hospital bed and the cost of hospital equipment for the bed)

i: Interest rate

N: The number of periods in the planning horizon.

According to the features of the hospital, the reception system, and the considered assumption in the proposed model, the queuing model *(M/M/m)* is the best model for the hospital system. Since the hospital beds are considered for the "servers" of the simulated queuing model of the hospital system, the queuing system used for the hospital is the model *(M/M/k).*
The aim is to determine optimal number of hospital beds (k^*). For this result, two different costs have been combined into an objective function:

$$Min\ C_T = \left[\frac{i.(1+i)^N}{(1+i)^N-1}\right].\sum_{n=0}^{K}(K-n).\pi_n.b + \sum_{n=K+1}^{\infty}(n-K).\pi_n.P \quad (1)$$

The first cost is caused by excess capacity ($k > k^*$). It means that if a hospital is constructed with excess capacity, this capacity would be left unused most of the time. Therefore, by constructing too big a hospital, excessive cost would be imposed on the investors and a hospital with such a high capacity would not be efficient. This cost would be referred to as Cost of Excess Capacity and is obtained from equation 2.

$$\sum_{n=0}^{K_j}\left(K_j-n\right).\pi_{nj}.b_j \quad (2)$$

The second cost is caused by the construction of a hospital with capacity less than optimal capacity ($k^* > k$). If a hospital with too small a capacity is constructed, then the hospital would be at full capacity most of the time and the hospital would not be able to meet the demands of patients. This cost would be referred to as Cost of Capacity Shortages and is obtained in equation 3.

$$\sum_{n=K_j+1}^{\infty}\left(n-K_j\right).\pi_{nj}.P_j \quad (3)$$

By changing hospital capacity, these two costs move in opposite directions to one another. Cost of Excess Capacity increases and Cost of Capacity Shortages decreases by increasing capacity, and vice versa (Figure 1).

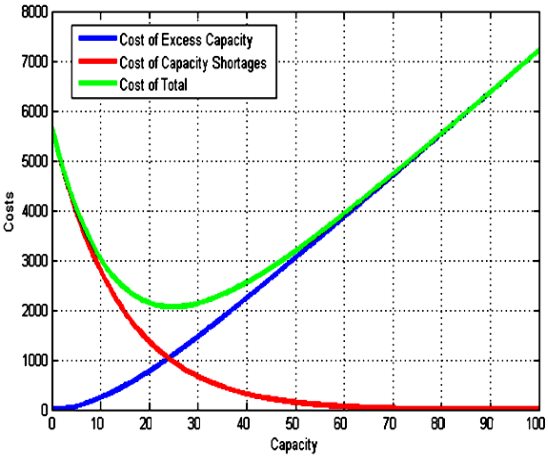

Fig. 1. The relation between costs and capacity

The total Cost of constructing a hospital with capacity *k* is equal to the sum of the Cost of Excess Capacity and the Cost of Capacity Shortages.
The cost of constructing additional capacity would be imposed on investors for a single time: that being the construction of the hospital. The cost of the capacity shortage, however, is incurred daily, over the life of the hospital. To be able to compare and combine these two costs, the cost of excess capacity should be distributed along time by considering time value of money. To get this result, the cost of excess capacity is multiplied by Capital-Recovery Factor (CRF).

$$CRF=(A/P,i,N)=[(i.(1+i)^N)/((1+i)^N-1)] \quad (4)$$

Values of π_n were obtained using queuing model of (*M/M/* k) (Gelenbe et al., 1987).

$$\pi_n=\begin{cases}\left(\frac{\lambda}{\mu}\right)^n\frac{\pi_0}{n!} & ; \quad n<k\\ \left(\frac{\lambda}{\mu}\right)^n\frac{\pi_0 k^{k-n}}{k!} & ; \quad n\ge k\end{cases} \quad (5)$$

$$\pi_{(0)}=[1+\sum_{(n=1)}^{(k-1)}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{n!}+\sum_{n=k}^{\infty}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{k!}\times\frac{1}{k^{n-k}}]^{-1} \quad (6)$$

In the long term, *i* and *P* are changeable and not predictable; therefore, an exact estimate of their values is not possible. As a result, these values are entered into the model with the help of fuzzy triangular estimates based on experts' opinions and experience at hand. Subscripts *L, M,* and *R* indicate the most pessimistic, the most probable, and the most optimistic values of the corresponding fuzzy triangular parameter, respectively.

$$\tilde{i}=\left(i_L,i_M,i_R\right) \qquad \tilde{P_j}=\left(P_L,P_M,P_R\right)$$

Therefore, in uncertain conditions and by considering the fuzzy triangular number calculations, the mathematical model is extended as in equations (7-11).

$$Min\tilde{C}_T=\left[\left[\frac{\tilde{i}.\left(1+\tilde{i}\right)^N}{\left(1+\tilde{i}\right)^N-1}\right].\sum_{n=0}^{K}(K-n).\pi_n b+\sum_{n=K+1}^{\infty}(n-K).\pi_n.\tilde{P}\right] \quad (7)$$

$$\tilde{C}_T=\left[\frac{\tilde{\imath}.(1+\tilde{\imath})^N}{(1+\tilde{\imath})^N-1}\right].\sum_{n=0}^{k}(k-n).\pi_n.b+\sum_{n=k+1}^{\infty}(n-k).\pi_n.\tilde{P}=(c_L,c_M,c_R) \quad (8)$$

$$c_L=\left[\frac{i^L.(1+i^L)^N}{(1+i^L)^N-1}\right].b.\sum_{n=0}^{k}(k-n).\pi_n+P_L.\sum_{n=k+1}^{\infty}(n-k).\pi_n \quad (9)$$

$$c_M=\left[\frac{i^M.(1+i^M)^N}{(1+i^M)^N-1}\right].b.\sum_{n=0}^{k}(k-n).\pi_n+P_M.\sum_{n=k+1}^{\infty}(n-k).\pi_n \quad (10)$$

$$c_R=\left[\frac{i^R.(1+i^R)^N}{(1+i^R)^N-1}\right].b.\sum_{n=0}^{k}(k-n).\pi_n+P_R.\sum_{n=k+1}^{\infty}(n-k).\pi_n \quad (11)$$

This cost function can be used to determine the optimal number of beds in different wards of the hospital operating under different conditions. To get this result, firstly, an appropriate queuing system must be defined with the consideration of the specific conditions of each department. Afterwards, π_n should be calculated for the defined system with the use Markov chains and equilibrium equations. Finally, the value of the total cost function ($\tilde{C}_T$) for various hospital capacities (*k*) should be calculated and the case that has minimum total cost would be the optimal hospital capacity.

3.2. Ranking fuzzy numbers

Due to fuzzy total cost function in the previous section, a fuzzy ranking technique is required to determine the minimum value of cost function and the optimal hospital capacity. There are numerous fuzzy ranking techniques (Asady, 2010; Asady & Zendehnam, 2007; Cheng, 1998; Chu & Tsao, 2002; Nejad & Mashinchi, 2011; Wang & Lee, 2008). In this article, Lee and Li's technique (E. S. Lee & Li, 1988) is used. In this approach, two criteria are used, namely mean and standard deviation. Fuzzy number's mean and standard deviation are obtained from equations 11 and 12.

$$\bar{X}\left(\tilde{M}\right)=\frac{\int_{s(\tilde{M})}x\left(\mu_{\tilde{M}}(x)\right)^{2}dx}{\int_{s(\tilde{M})}\left(\mu_{\tilde{M}}(x)\right)^{2}dx} \quad (12)$$

$$\delta\left(\tilde{M}\right)=\left[\frac{\int_{s(\tilde{M})}x^{2}\left(\mu_{\tilde{M}}(x)\right)^{2}dx}{\int_{s(\tilde{M})}\left(\mu_{\tilde{M}}(x)\right)^{2}dx}-\left(\bar{X}\left(\tilde{M}\right)\right)^{2}\right]^{\frac{1}{2}} \quad (13)$$

Given triangular fuzzy numbers, their mean and standard deviation are obtained from equations 13 and 14.

$$\bar{X}\left(\tilde{M}\right)=\frac{1}{4}(l+2m+n) \quad (14)$$

$$\delta\left(\tilde{M}\right)=\frac{1}{80}(3l^{2}+4m^{2}+3n^{2}-2nl-4lm-4mn) \quad (15)$$

The prioritization rule for ranking two fuzzy numbers $\tilde{M}_i$ and $\tilde{M}_j$ is depicted in table 1.

4. Case Study

Razi hospital is located next to the passenger terminal in the city of Torbat Heydarieh. The hospital began its activities on January, 1996. The hospital area is 14 thousand square meters. The hospital is operated by 250 medical, administrative, and service staff. The hospital has inpatient, outpatient, and para-clinic departments. Each hospital department includes a variety of clinics and hospital units. Emergency unit is a subset of the hospital outpatient department.

Table 1
Ranking fuzzy numbers

Comparison of mean values	Comparison of sd values	Prioritization result
$\bar{X}(\tilde{M}_i) > \bar{X}(\tilde{M}_j)$	————	$\tilde{M}_i > \tilde{M}_j$
$\bar{X}(\tilde{M}_i) = \bar{X}(\tilde{M}_j)$	$\sigma(\tilde{M}_i) < \sigma(\tilde{M}_j)$	$\tilde{M}_i > \tilde{M}_j$

Since Torbat Heydarieh is close to Mashhad, Torbat - Mashhad road is the most important route for Mashhad travelers. In recent years, increases in passenger traffic have caused an increase in accidents, and consequently increase patient referrals to the emergency unit of the Razi hospital. Therefore, in September 2013, capacity increase of the emergency unit of the hospital was approved. In late 2014, the hospital began research projects in collaboration with the University of Torbat Heydarieh in order to determine the optimal capacity of emergency unit. A brief summary of the research project is reported in this paper.

4.1. Numerical results for emergency unit of Razi hospital

Suppose that patients arrive at the emergency room according to a Poisson process with an average of 22 patients per day. The lengths of stay of the patients on hospital beds in the emergency department have the exponential random variable with a mean of 0.5 days. Assume that the cost of creation and increase of capacity by a single bed in the emergency department is 1,500 currency (currency to be considered); that revenue from the hospitalization per patient per day is approximately 50 currency and in the 120-month planning horizon, a monthly interest rate of approximately 2% is assumed. What is the optimal number of hospital beds available for allocation to the emergency unit of the hospital?
According to the expert opinions, imprecise parameters are displayed as in the following triangular fuzzy numbers.

$\tilde{\imath}$ = (1.5%, 2%, 4%)

$\tilde{P}$ = (40, 50, 55)

4.2. Solution

Using the code written in MATLAB, the calculations of fuzzy values for the proposed cost function for different capacities and their results are summarized in Table 2.
It is worth noting that in the MATLAB code, the second summation's upper bound must be substituted by a large number rather than by infinity. This large number must be chosen such that the addition of probabilities equals almost one. In this example, we have chosen an upper bound equal to 100.

$$\sum_{n=0}^{100}\pi_n = 0/9998$$

Table 2.
Fuzzy values of the cost function

K	$\tilde{C}_T$
0	(2010.10, 2334.14, 3255.67)
1	(2000.25, 2108.14, 3250.00)
2	(1894.99, 1910.24, 3001.84)
3	(1680.99, 1721.24, 2655.84)
.	.
97	(6732.50, 7541.61, 11850.44)
98	(6742.53, 7551.63, 11866.85)
99	(6750.03, 7560.03, 11880.05)
100	(6757.53, 7568.43, 11893.25)

By using the code written in MATLAB software for Lee and Li method, fuzzy numbers for the cost function and capacities range from the lowest to the highest numbers are reported in table 3.
It is evident from Table 3 that by constructing a hospital with a capacity of 18, the total cost would be at the lowest level possible, hence this capacity is chosen as optimal. If, for any reason, the mentioned capacity could not be constructed, 19, 17, and more would be the next most preferable options.

It is worth noting that the amount of parameters i, μ, λ, b, p was influential in determining the best capacity. Therefore, a sensitivity analysis of input parameters is needed to identify sensitive and insensitive parameters. It is obvious that more attention should be paid to estimate the sensitive parameters

Table 3.
Prioritization of different capacities based on fuzzy cost function

Priorities based on cost	$\tilde{C}_T$	K
1	(1000.10, 1464.14, 2249.67)	18
2	(1001.25, 1467.14, 2250.65)	19
3	(1003.99, 1469.24, 2252.84)	17
.	.	.
99	(6742.53, 7551.63, 11866.85)	98
100	(6750.03, 7560.03, 11880.05)	99
101	(6757.53, 7568.43, 11893.25)	100

Table 4.
Sensitivity analysis of parameter i

percentage change in the parameter i	Prioritize top 20 scenarios	The amount of change
0%	$k_{18} > k_{19} > k_{17} > k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{16} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{15} > k_{31} > k_{32} > k_{33} > k_{34}$	-
+10%	$k_{18} > k_{19} > k_{17} > k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{16} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{15} > k_{31} > k_{32} > k_{33} > k_{34}$	-
+20%	$k_{18} > k_{19} > k_{17} > k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{16} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{15} > k_{31} > k_{32} > k_{33} > k_{34}$	-
+30%	$k_{18} > k_{19} > k_{17} > k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{16} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{15} > k_{31} > k_{32} > k_{33} > k_{34}$	-
+40%	$k_{18} > k_{19} > k_{17} > k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{16} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{15} > k_{31} > k_{32} > k_{33} > k_{34}$	-
+50%	$k_{17} > k_{18} > k_{19} > k_{16} > k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{26} > k_{15} > k_{27} > k_{28} > k_{14} > k_{29} > k_{30} > k_{31} > k_{32} > k_{33}$	**low**

Table 5.
Sensitivity analysis of parameter λ

percentage change in the parameter λ	Prioritize top 20 scenarios	The amount of change
0%	$k_{18} > k_{19} > k_{17} > k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{16} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{15} > k_{31} > k_{32} > k_{33} > k_{34}$	-
+10%	$k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{19} > k_{25} > k_{26} > k_{27} > k_{28} > k_{29} > k_{18} > k_{30} > k_{31} > k_{32} > k_{33} > k_{34} > k_{17} > k_{35} > k_{36}$	**Medium**
+20%	$k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{19} > k_{26} > k_{27} > k_{28} > k_{29} > k_{18} > k_{30} > k_{31} > k_{32} > k_{33} > k_{34} > k_{35} > k_{36} > k_{37}$	**Medium**
+30%	$k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{26} > k_{20} > k_{27} > k_{28} > k_{29} > k_{30} > k_{31} > k_{19} > k_{32} > k_{33} > k_{34} > k_{35} > k_{36} > k_{37} > k_{38}$	**High**
+40%	$k_{23} > k_{24} > k_{25} > k_{22} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{31} > k_{32} > k_{21} > k_{33} > k_{34} > k_{35} > k_{36} > k_{37} > k_{38} > k_{39} > k_{40}$	**High**
+50%	$k_{25} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{24} > k_{31} > k_{32} > k_{33} > k_{34} > k_{35} > k_{36} > k_{23} > k_{37} > k_{38} > k_{39} > k_{40} > k_{41} > k_{42}$	**High**

Table 6
Sensitivity analysis of parameter μ

percentage change in the parameter μ	Prioritize top 20 scenarios	The amount of change
0%	$k_{18} > k_{19} > k_{17} > k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{16} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{15} > k_{31} > k_{32} > k_{33} > k_{34}$	-
+10%	$k_{17} > k_{18} > k_{19} > k_{20} > k_{16} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{15} > k_{26} > k_{27} > k_{28} > k_{14} > k_{29} > k_{30} > k_{31} > k_{32} > k_{33}$	**Low**
+20%	$k_{16} > k_{17} > k_{18} > k_{19} > k_{20} > k_{15} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{14} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{13} > k_{31} > k_{32}$	**Medium**
+30%	$k_{14} > k_{15} > k_{16} > k_{17} > k_{13} > k_{18} > k_{19} > k_{20} > k_{21} > k_{12} > k_{22} > k_{23} > k_{24} > k_{25} > k_{26} > k_{27} > k_{28} > k_{11} > k_{29} > k_{30}$	**High**
+40%	$k_{13} > k_{14} > k_{15} > k_{16} > k_{17} > k_{18} > k_{12} > k_{19} > k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{11} > k_{25} > k_{26} > k_{27} > k_{28} > k_{29} > k_{10}$	**High**
+50%	$k_{11} > k_{12} > k_{13} > k_{14} > k_{15} > k_{16} > k_{10} > k_{17} > k_{18} > k_{19} > k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{9} > k_{25} > k_{26} > k_{27} > k_{28}$	**High**

Table 7
Sensitivity analysis of parameter b

percentage change in the parameter b	Prioritize top 20 scenarios	The amount of change
0%	$k_{18} > k_{19} > k_{17} > k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{16} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{15} > k_{31} > k_{32} > k_{33} > k_{34}$	-
+10%	$k_{18} > k_{19} > k_{17} > k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{16} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{15} > k_{31} > k_{32} > k_{33} > k_{34}$	-
+20%	$k_{18} > k_{19} > k_{17} > k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{16} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{15} > k_{31} > k_{32} > k_{33} > k_{34}$	-
+30%	$k_{18} > k_{19} > k_{17} > k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{16} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{15} > k_{31} > k_{32} > k_{33} > k_{34}$	-
+40%	$k_{17} > k_{18} > k_{19} > k_{20} > k_{21} > k_{22} > k_{16} > k_{23} > k_{24} > k_{25} > k_{26} > k_{15} > k_{27} > k_{28} > k_{29} > k_{30} > k_{14} > k_{31} > k_{32} > k_{33}$	**low**
+50%	$k_{16} > k_{17} > k_{18} > k_{19} > k_{15} > k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{14} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{31} > k_{32}$	**low**

Table 8.
Sensitivity analysis of parameter p

percentage change in the parameter p	Prioritize top 20 scenarios	The amount of change
0%	$k_{18} > k_{19} > k_{17} > k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{16} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{15} > k_{31} > k_{32} > k_{33} > k_{34}$	**-**
+10%	$k_{18} > k_{19} > k_{17} > k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{16} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{15} > k_{31} > k_{32} > k_{33} > k_{34}$	**-**
+20%	$k_{19} > k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{18} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{17} > k_{31} > k_{32} > k_{33} > k_{34} > k_{35} > k_{16}$	**Low**
+30%	$k_{20} > k_{21} > k_{22} > k_{23} > k_{19} > k_{24} > k_{25} > k_{26} > k_{27} > k_{28} > k_{18} > k_{29} > k_{30} > k_{31} > k_{32} > k_{17} > k_{33} > k_{34} > k_{35} > k_{36}$	**Medium**
+40%	$k_{20} > k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{19} > k_{26} > k_{27} > k_{28} > k_{29} > k_{30} > k_{18} > k_{31} > k_{32} > k_{33} > k_{34} > k_{35} > k_{36} > k_{37}$	**Medium**
+50%	$k_{21} > k_{22} > k_{23} > k_{24} > k_{25} > k_{26} > k_{27} > k_{28} > k_{20} > k_{29} > k_{30} > k_{31} > k_{32} > k_{33} > k_{19} > k_{34} > k_{35} > k_{36} > k_{37} > k_{38}$	**High**

4.3. Sensitivity analysis

Tables 4-8 show the MATLAB output results of the sensitivity analysis for each input parameters of the problem. In these calculations, each parameter is increased by 10% in each step separately. Afterwards, priority changes of the top 20 scenarios were studied.

Analyzing the results in Table 4 indicates that by increasing (I), the final ranking of the scenarios does not change substantially. According to Table 4, this parameter is not a sensitive parameter of the model because only a change greater than 50 percent causes the final ranking to change.

The results of the sensitivity analysis for parameter λ in Table 5 indicate that this parameter is a sensitive parameter of the model, and it has a significant effect on the final ranking of the scenarios. As could be expected, increasing λ causes the model to select more capacity for the emergency unit because by increasing λ, the rate of patients' arrivals to hospital is increased so that more beds are needed to deal with the situation. Care should be taken while estimating λ, because this parameter is a sensitive parameter of the model.

Table 6 indicates the results of the sensitivity analysis for parameter μ. Increasing μ means duration of hospitalization (1/μ) is reduced. Logically, by reducing the duration of hospitalization, fewer beds are needed. According to Table 6, this parameter is a sensitive parameter of the model.

Table 7 indicates the results of the sensitivity analysis for parameter (b). This parameter affects the amounts of cost of excess capacity. Also, cost of excess capacity affects the final cost considering the formula (1). Therefore, parameter (b) affects the final cost of the hospital indirectly. But, according to Table 7, this parameter is an insensitive parameter of the model.

Sensitivity analysis of parameter p in Table 8 indicates that although this parameter could not be assumed as sensitive as λ, but accurate estimation of this parameter is important (in comparison with i , b), because it has caused drastic changes in the final ranking in some cases. Also, increasing this parameter resulted in higher priority for those scenarios with more assigned number of beds in the final ranking.

5. Conclusion and Further Studies

In this paper, the well-known problem of determining the optimal capacity of hospital was addressed. For this purpose, the system of hospitalization has been modeled following a queuing system. With consideration given to the costs of capacity shortage and the possibility of excess capacity, an unrestricted model for the problem was developed. In order to adapt the models to real conditions, some of the parameters of the problem were considered as fuzzy numbers. Considering the mentioned matters and the example, it can be concluded that to obtain the optimal hospital capacity using queuing theory concepts, one must determine a queuing model that matches the hospital conditions and calculate π_n according to the queuing model. Various $\pi_n$values are placed in the cost function. The cost function value is calculated for various hospital capacities and the capacity that allocates the minimum value of cost function to itself is chosen as the optimal hospital capacity. In the mentioned case study, m/m/k model is used to model hospital's emergency unit. In accordance to results of table 3, 18 hospital beds are required in order to minimize the hospital's total costs. Furthermore, tables 4 through 8 show that λ, μ, and p are sensitive parameters, and it is important to estimate them carefully. The proposed approach in this article is applicable to any queuing model. Considering the flexibility of Markovian chains and the various queuing models, the approach makes it possible to model various hospital units with different conditions. For example, in some hospital units such as operating rooms, patients' reference to beds or duration of hospitalization may not follow the functions of Poisson and exponential. In these cases, general functions can be used, such as (G / M / 1), (G / G / m) or ($G^{[r]}$/ G /1). Furthermore, there are more complex hospital models whose model does not match the queuing model completely. For such cases, it is suggested that a combination of these models and simulations be used. In the real world, arrival rate and hospitalization rate are not certain. By considering them as fuzzy and using fuzzy probabilities, π_n values can be calculated and entered into the model.

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References

Asady, B. (2010). The revised method of ranking LR fuzzy number based on deviation degree. *Expert Systems with Applications, 37*(7), 5056-5060.

Asady, B, & Zendehnam, A. (2007). Ranking fuzzy numbers by distance minimization. *Applied Mathematical Modelling, 31*(11), 2589-2598.

Bazzoli, Gloria J, Brewster, Linda R, Liu, Gigi, & Kuo, Sylvia. (2003). Does US hospital capacity need to be expanded? *Health Affairs, 22*(6), 40-54.

Ben Bachouch, Rym, Guinet, Alain, & Hajri-Gabouj, Sonia. (2012). An integer linear model for hospital bed planning. *International Journal of Production Economics, 140*(2), 833-843. doi: http://dx.doi.org/10.1016/j.ijpe.2012.07.023

Cheng, Ching-Hsue. (1998). A new approach for ranking fuzzy numbers by distance method. *Fuzzy sets and systems, 95*(3), 307-317.

Chu, Ta-Chung, & Tsao, Chung-Tsen. (2002). Ranking fuzzy numbers with an area between the centroid point and original point. *Computers & Mathematics with Applications, 43*(1), 111-117.

Cochran, Jeffery K., & Roche, Kevin T. (2009). A multi-class queuing network analysis methodology for improving hospital emergency department performance. *Computers & Operations Research, 36*(5), 1497-1512. doi: http://dx.doi.org/10.1016/j.cor.2008.02.004

Coile, Russell C, & Association, American Hospital. (2002). *Futurescan 2002: A Forecast of Healthcare Trends, 2002-2006*. Chicago: Health Administration Press Chicago.

Crandall, R. E., & Markland, R. E. (1996). DEMAND MANAGEMENT-TODAY'S CHALLENGE FOR SERVICE INDUSTRIES. *Production and Operations Management, 5*(2), 106-120. doi: 10.1111/j.1937-5956.1996.tb00389.x

Garg, L., McClean, S., Meenan, B., & Millard, P. (2010). A non-homogeneous discrete time Markov model for admission scheduling and resource planning in a cost or capacity constrained healthcare system. *Health Care Management Science, 13*(2), 155-169. doi: 10.1007/s10729-009-9120-0

Gelenbe, E., Pujolle, G., & Nelson, JCC. (1987). *Introduction to queueing networks* (2th ed.): Wiley Chichester.

Getz, D. (1983). Capacity to absorb tourism: Concepts and implications for strategic planning. *Annals of Tourism Research, 10*(2), 239-263. doi: http://dx.doi.org/10.1016/0160-7383(83)90028-2

Gong, Y., Zhang, J., & Fan, Z. (2010). *A multi-objective comprehensive learning particle swarm optimization with a binary search-based representation scheme for bed allocation problem in general hospital*. Paper presented at the Systems Man and Cybernetics (SMC), Istanbul.

Gorunescu, F., McClean, S., & Millard, P.H. (2002). Using a Queueing Model to Help Plan Bed Allocation in a Department of Geriatric Medicine. *Health Care Management Science, 5*(4), 307-312. doi: 10.1023/A:1020342509099

Gu, Z. (2003). Analysis of Las Vegas strip casino hotel capacity: an inventory model for optimization. *Tourism Management, 24*(3), 309-314.

Hershey, J. C., Weiss, E. N, & Cohen, M. A. (1981). A stochastic service network model with application to hospital facilities. *Operations Research, 29*(1), 1-22.

Hwang, J., Gao, L., & Jang, W. (2010). Joint demand and capacity management in a restaurant system. *European Journal of Operational Research, 207*(1), 465-472. doi: http://dx.doi.org/10.1016/j.ejor.2010.04.001

Kao, E., & Tung, G.. (1981). Bed allocation in a public health care delivery system. *Management Science, 27*(5), 507-520.

Klassen, K. J., & Rohleder, T. R. (2002). Demand and capacity management decisions in services: how they impact on one another. *International Journal of Operations & Production Management, 22*(5), 527-548.

Kokangul, A.. (2008). A combination of deterministic and stochastic approaches to optimize bed capacity in a hospital unit. *Computer methods and programs in biomedicine, 90*(1), 56-65.

Lapierre, S. D., Goldsman, D., Cochran, R., & DuBow, J. (1999). Bed allocation techniques based on census data. *Socio-Economic Planning Sciences, 33*(1), 25-38. doi: http://dx.doi.org/10.1016/S0038-0121(98)00008-1

Lee, Donald KK, & Zenios, Stefanos A. (2009). Optimal capacity overbooking for the regular treatment of chronic conditions. *Operations research, 57*(4), 852-865.

Lee, E. S., & Li, R. J. (1988). Comparison of fuzzy numbers based on the probability measure of fuzzy events. *Computers & Mathematics with Applications, 15*(10), 887-896. doi: http://dx.doi.org/10.1016/0898-1221(88)90124-1

Li, Ling, & Benton, W. C. (2003). Hospital capacity management decisions: Emphasis on cost control and quality enhancement. *European Journal of Operational Research, 146*(3), 596-614. doi: http://dx.doi.org/10.1016/S0377-2217(02)00225-4

Li, X., Beullens, P., Jones, D., & Tamiz, M. (2008). An integrated queuing and multi-objective bed allocation model with application to a hospital in China. *J Oper Res Soc, 60*(3), 330-338.

McManus, M. L., Long, M. C., Cooper, A., & Litvak, E. (2004). Queuing theory accurately models the need for critical care resources. *Anesthesiology, 100*(5), 1271-1276.

Milne, E., & Whitty, P. (1995). Calculation of the need for paediatric intensive care beds. *Archives of disease in childhood, 73*(6), 505-507.

Nejad, A. M., & Mashinchi, Mashaallah. (2011). Ranking fuzzy numbers based on the areas on the left and the right sides of fuzzy number. *Computers & Mathematics with Applications, 61*(2), 431-442.

Pullman, M., & Rodgers, S. (2010). Capacity management for hospitality and tourism: A review of current approaches. *International Journal of Hospitality Management, 29*(1), 177-187.

Taylor, G. D. (1980). How to match plant with demand: a matrix for marketing. *International Journal of Tourism Management, 1*(1), 56-60.

Walczak, S., Pofahl, W. E., & Scorpio, Ronald J. (2003). A decision support tool for allocating hospital bed resources and determining required acuity of care. *Decision Support Systems, 34*(4), 445-456. doi: http://dx.doi.org/10.1016/S0167-9236(02)00071-4

Wang, Y., & Lee, H. (2008). The revised method of ranking fuzzy numbers with an area between the centroid and original points. *Computers & Mathematics with Applications, 55*(9), 2033-2042.

White, J. A, & Francis, Richard L. (1971). Normative models for some warehouse sizing problems. *AIIE Transactions, 3*(3), 185-190.

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